Chapter 3

Exercise 3.1

v has access to no world. So any sentence A is true at *all* (zero) worlds accessible from v.

If this seems strange, remember that $\Box A$ is equivalent to $\neg\Diamond\neg A$. And $\Diamond\neg A$ means that there's an accessible world where $\neg A$ is true. If there are no accessible worlds, then this is false. So $\neg\Diamond\neg A$ is true.

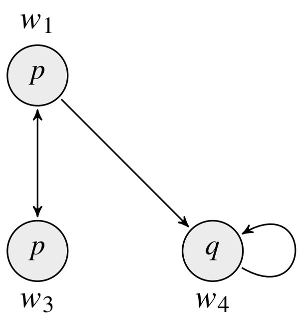
Exercise 3.2

(a) w_1, w_2, and w_3; (b) w_3; (c) –; (d) w_1, w_2 and w_4; (e) all.

Exercise 3.3

There are infinitely many correct answers for each world. For example: $w_1 : \Diamond\Box p$, $w_2 : \neg p \land \neg q$, $w_3 : \Box p$, $w_4 : \Box q$.

Exercise 3.4



Exercise 3.5

(a) For example: $W = \{w, v\}$, $R = \{(w, v), (v, w)\}$, $V(p) = \{v\}$. $\Box p \to \Box\Box p$ is false at w. ('$R = \{(w, v), (v, w)\}$' means that R relates w to v and v to w and nothing else to anything else.)

(b) For example: $W = \{w, v\}$, $R = \{(w, w), (w, v)\}$, $V(p) = \{w\}$. $\Diamond p \to \Box\Diamond p$ is false at w.

Exercise 3.6

For example: $\Box(p \vee \neg p) \to (p \vee \neg p)$.

Exercise 3.7

By clause (g) of definition 3.2, $\Box(p \vee \neg p)$ is false at a world w in a Kripke model only if $p \vee \neg p$ is false at some world accessible from w. By clause (d) of definition 3.2, $p \vee \neg p$ is false at a world only if both p and $\neg p$ are false at the world, which by clause (a) means that p is both true and false at the world. This is impossible. So $\Box(p \vee \neg p)$ is not false at any world in any Kripke model.

Exercise 3.8

By definition 3.2, $\Box p \to \Diamond p$ is false at a world w in a Kripke model only if $\Box p$ is true at w and $\Diamond p$ is false at w. But if w has access to itself then the truth of $\Box p$ at w implies that p is true at w, and then $\Diamond p$ is false at w. So $\Box p \to \Diamond p$ can't be false at any world in any Kripke model in which each world has access to itself.

Exercise 3.9

Reflexive yes, serial yes, transitive yes, euclidean no, symmetric no, universal no.

Exercise 3.10

(a) Suppose R is symmetric and transitive, and that xRy and xRz. By symmetry, yRx. By transitivity, yRz.
(b) Suppose R is symmetric and euclidean, and that xRy and yRz. By symmetry, yRx. By euclidity, xRz.
(c) Suppose R is reflexive and euclidean, and that xRy. By reflexivity, yRy. By euclidity, yRx.

Exercise 3.11

It's true that if R is symmetric and transitive then wRv implies vRw which implies wRw. But this only shows that every world w *that can see some world* v can see itself. Symmetry, transitivity, *and seriality* together imply reflexivity. Symmetry and transitivity alone do not.

Exercise 3.12

(a) Every world has access only to itself.
(b) No world has access to any world.

Exercise 3.13

You can enter the sentences at wolfgangschwarz.net/trees. To check for K-validity, leave all the checkboxes (for 'universal' etc.) empty.

Exercise 3.14

You can enter the sentences at wolfgangschwarz.net/trees. To test for K4-validity, check the 'transitive' box. To test for D-validity, check 'serial'. To test for B-validity, check 'symmetric'. To test for T-validity, check 'reflexive'.