Chapter 6

Exercise 6.1

(a) $\mathsf{O}\,\neg p$; p: You go into the garden.
(b) $\mathsf{O}\,\neg p$; p: You go into the garden.
(c) $\mathsf{O}\,p$; p: Jones helps his neighbours.
(d) $\mathsf{O}(p \to q)$; p: Jones helps his neighbours, q: Jones tells his neighbours that he's coming.
(e) You might try $\mathsf{O}(\neg p \to \neg q)$ or $\neg p \to \mathsf{O}\,\neg q$ p: Jones helps his neighbours, q: Jones tells his neighbours that he's coming.

See section 6.3, especially exercise 6.13, for why neither translation of (e) is fully satisfactory.

Exercise 6.2

(a): $\Box(\mathsf{N} \to (\Box(\mathsf{N} \to A) \to A))$. (b): use umsu.de/trees/.

Exercise 6.3

$\mathsf{P}\,A$ could be defined as $\neg\Box(\mathsf{N} \to \neg A)$, or more simply (and equivalently) as $\Diamond(\mathsf{N} \land A)$.

Exercise 6.4

Transitivity (if wRv and vRu then wRu) and euclidity (if wRv and wRu then vRu) both state that if v is ideal and u is ideal then u is ideal.

Exercise 6.5

R is euclidean if $\forall x \forall y \forall z((xRy \land xRz) \to yRz)$. Suppose wRv. Instantiating the universal formula with w for x and with v for y and z, we have $(wRv \land wRv) \to vRv$. So vRv.

Exercise 6.6

Consider the example from the text, where w is the actual world (in the UK) and u is a w-accessible world at which everyone drives on the left although the law says that one must drive on the right. A typical world accessible from u will be a world at which people drive on the right. This world will not be accessible from w. So we have a counterexample to transitivity. We also have a counterexample to euclidity because we have wRu and wRu but not uRu. (Euclidity entails shift reflexivity.)

Exercise 6.7

Use https://www.umsu.de/trees/. (Write O as a box and P as a diamond. For D, make the accessibility relation serial; for KD45, make it serial, transitive, and euclidean.)

Exercise 6.8

(Dual1) says that $\neg\Diamond A$ is equivalent to $\Box\neg A$. If nothing is permitted then $\neg\Diamond A$ is true for all A. But if nothing is forbidden then $\Box\neg A$ is false for all A.

(Dual2) says that $\neg\Box A$ is equivalent to $\Diamond\neg A$. If nothing is forbidden then $\neg\Box A$ is true for all A. But if nothing is permitted then $\Diamond\neg A$ is false for all A.

Exercise 6.9

In the described situation, it ought to be the case that Amy is either obligated to help Betty or obligated to help Carla, but Amy is neither obligated to help Betty nor to help Carla. So if p translates 'Amy helps Betty' and q 'Amy helps Carla', we seem to have $\mathsf{O}(\mathsf{O}\,p \vee \mathsf{O}\,q)$ and $\neg\,\mathsf{O}\,p$ and $\neg\,\mathsf{O}\,q$. But these assumptions are inconsistent in K5. You can draw a K5-tree (using the K-rules and the Euclidity rule) starting with $\mathsf{O}(\mathsf{O}\,p \vee \mathsf{O}\,q)$ and $\neg\,\mathsf{O}\,p$ and $\neg\,\mathsf{O}\,q$ on which all branches close. This shows that there is no world in any euclidean model at which the three assumptions are true.

Exercise 6.10

Since we assume that there is always at least one best world among the accessible worlds, and the accessible worlds comprise just one world, it follows that $\mathsf{O}\,A$ is true at w iff A is true at w. The logic we get is the "Triv" logic that is axiomatized by adding the (Triv)-schema $\Box A \leftrightarrow A$ to the standard axioms and rules for K. This logic is stronger than S5: all S5-valid sentences are Triv-valid. (We also have, among other things, all instances of $\Box A \leftrightarrow \Diamond A$.) The choice between absolutism and relativism makes no difference.

Exercise 6.11

Use umsu.de/trees/.

Exercise 6.12

Deontic detachment is valid. Suppose A is true at the best of the (circumstantially) accessible worlds, and B is true at the best of the accessible worlds at which A is true. Then B is true at the best of the accessible worlds.

Factual detachment is invalid. A counterexample is the "gentle murder puzzle". Suppose John is determined to kill his grandmother. *If he will go ahead and kill her, he ought to do so gently*. Can we conclude that John ought to gently kill his grandmother? Arguably not. He shouldn't kill her at all! We have k and $\mathsf{O}(g/k)$, but not $\mathsf{O}(g)$. Formally, g is true at the best of the accessible k-worlds, but since all the k-worlds are quite bad, g is not true at the best of the accessible worlds.

Exercise 6.13

(c) can obviously be translated as $\mathsf{O}\, p$, (f) as $\neg p$.

You probably translated (d) as either $p \to \mathsf{O}\, q$ or as $\mathsf{O}(p \to q)$. $p \to \mathsf{O}\, q$ is entailed by (f). The translation can't be right because it is easy to think of a scenario in which (f) is true but (d) false. Assume then that (d) is translated as $\mathsf{O}(p \to q)$.

The most obvious translations for (e) are $\neg p \to \mathsf{O}\, \neg q$ and $\mathsf{O}(\neg p \to \neg q)$. The latter is entailed by (c). But it is easy to think of a scenario in which (c) is true but (e) false. If (e) is translated as $\neg p \to \mathsf{O}\, \neg q$, then (c)–(f) constitute a deontic dilemma: (e) and (f) would entail $\mathsf{O}\, \neg t$, but (c) and (d) would entail $\mathsf{O}\, t$.

Exercise 6.14

Simply replace 'all' in the semantics for $\mathsf{O}(B/A)$ with 'some'.

Exercise 6.15

Ross's Paradox: 'Alice must be in the office or in the library' seems to imply that Alice might be in the office and that she might be in the library.

The Paradox of Free Choice: 'Alice might be in the office or in the library' seems to imply that Alice might be in the office and that she might be in the library.

Exercise 6.16

For every world w, every member of $N(w)$ contains w.

Exercise 6.17

In Kripke semantics, $\Box p$ and $\Box q$ together entail $\Box(p \wedge q)$. But if the probability of p is above the threshold and the probability of q is above the threshold, it does not follow that the probability of $p \wedge q$ is above the threshold. For example, we could have $\Pr(p) = 0.95$, $\Pr(q) = 0.94$, and $\Pr(p \wedge q) = 0.95 \times 0.94 = 0.893$.

Exercise 6.18

A bad dart player may have the ability to hit the dart board but lack the ability to hit the left half of the board and also the ability to hit the right half of the board.