Chapter 10

Exercise 10.1

(a), (b), (c), (e), (f), (h) are true; (d), (g) are false.

Exercise 10.2

Use wolfgangschwarz.net/trees/. Note that the website uses slightly different identity rules: instead of the Self-Identity rule, it has a rule for closing any branch that contains a statement of the form $\tau \neq \tau$.

Exercise 10.3

(a) $W = \{w\}$, wRw, $D = \{\text{Alice}\}$, $V(F, w) = \{\text{Alice}\}$, $V(G, w) = \emptyset$
(b) $W = \{w, v\}$, wRw and wRv, $D = \{\text{Alice}, \text{Bob}\}$, $V(F, w) = \{\text{Alice}\}$, $V(F, v) = \{\text{Bob}\}$
(c) $W = \{w, v\}$, wRw and wRv, $D = \{\text{Alice}, \text{Bob}\}$, $V(F, w) = \{\text{Alice}\}$, $V(F, v) = \emptyset$
(d) $W = \{w, v\}$, wRw and wRv, $D = \{\text{Alice}, \text{Bob}\}$, $V(P, w) = \{\text{Alice}\}$, $V(P, v) = \emptyset$, $V(Q, w) = \{\text{Alice}\}$, $V(Q, v) = \emptyset$

Exercise 10.4

$\Box\forall x\exists y(x = y) \rightarrow \forall x\Box\exists y(x = y)$ is an instance of the Converse Barcan Formula. If we read the box as a relevant kind of circumstantial necessity, and Loafy could have failed to exist, the consequent of this conditional is false. But the antecedent is true.

Exercise 10.5

(1) is equivalent to the Barcan Formula, (4) to the Converse Barcan Formula. (2) is highly implausible. (1) and (4) are often regarded as implausible, for the reasons I discuss in the text. (3) is about as plausible or implausible as the Converse Barcan Formula.

Exercise 10.6

(a)

1.	$\exists x\Box Fx \to \Box\exists xFx$	(w)	(Ass.)
2.	$\exists x\Box Fx$	(w)	(1)
3.	$\neg\Box\exists xFx$	(w)	(1)
4.	$\Box Fa$	(w)	(2)
5.	wRv		(3)
6.	$\neg\exists xFx$	(v)	(3)
7.	Fa	(v)	(4,5)
8.	$a=a$	(v)	(7)

9.	$a\neq a$	(v)	(6)	9.	$\neg Fa$	(v)	(6)
	x				x		

(b) DIY. The tree has four branches. I can't typeset it.

(c)

1.	$\neg\Box\exists x\, x=x$	(w)	(Ass.)
2.	wRv		(1)
3.	$\neg\exists x\, x=x$	(v)	(1)
4.	$a=a$	(v)	(Ex.)

9.	$a\neq a$	(v)	(3)	9.	$a\neq a$	(v)	(3)
	x				x		

(d)

1.	$\neg(\Diamond Fa \to \Diamond\exists xFx)$	(w)	(Ass.)
2.	$\Diamond Fa$	(w)	(1)
3.	$\neg\Diamond\exists x\, Fx$	(w)	(1)
4.	wRv		(2)
5.	Fa	(v)	(2)
6.	$a=a$	(v)	(5)
7.	$\neg\exists x\, Fx$	(v)	(3,4)

9.	$a\neq a$	(v)	(3)	10.	$\neg Fa$	(v)	(3)
	x				x		

(e)

1.	$\neg(a=b \rightarrow \Box(a=a \rightarrow a=b))$	(w)	(Ass.)
2.	$a=b$	(w)	(1)
3.	$\neg\Box(a=a \rightarrow a=b)$	(w)	(1)
4.	wRv		(3)
5.	$\neg(a=a \rightarrow a=b)$	(v)	(3)
6.	$a=a$	(v)	(5)
7.	$\neg a=b$	(v)	(5)
8.	$a=b$	(v)	(2,6)
	x		

Exercise 10.7

In the definition of a model, we could allow the interpretation function to be undefined for some names. We might also allow the sets D_w to be empty. We could leave the truth definition as it is.

Exercise 10.8

In the Superman case, Clark Kent and Superman are the same person, but Lois Lane doesn't know that they are. So we appear to have $s=c$ but not $\Box(s=c)$. Similarly, in the Julius case, Julius and Whitcomb L. Judson are the same person, but one may well not know that they are. In the Goliath case, we have Lumpl = Goliath without it being metaphysically necessary that Lumpl = Goliath, as there are worlds in which Lumpl is a bowl and Goliath is not.

Exercise 10.9

We would assume that (i) the name g picks out a statue at all accessible worlds, (ii) l picks out a lump of clay at all accessible worlds, and (iii) at the actual world, l and g pick out the same thing: the statue-shaped lump on the shelf.

Exercise 10.10

The premises are $\Box\exists x(x = i)$ and $\neg\Box\exists x(x = b)$. The conclusion is $i \neq b$. The argument is CK-valid and VK-valid.

Exercise 10.11

Translation: $\exists x(Tx \wedge Wx \wedge \neg KWx \wedge \neg K\neg Wx)$, where T translates '– is a ticket' and '– will win'.

If variables are directly referential, then this sentence is true in any scenario in which I don't know which ticket will win.

Exercise 10.12

To render $\forall x \forall y(x = y \rightarrow \Box x = y)$ valid, we can restrict the eligible individual concepts in a model as follows. For any individual concepts f and g and worlds w and v, if wRv and $f(w) = g(w)$ then $f(v) = g(w)$. (We do not stipulate that if wRv and $f(v) = g(v)$ then $f(w) = g(w)$, which would render the necessity of distinctness valid.)