# Logic 2: Modal Logic

## Lecture 1

Wolfgang Schwarz <wolfgang.schwarz@ed.ac.uk>

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University of Edinburgh

# Course info

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- This is an intermediate logic course focusing on modal logic. You will learn about
  - the syntax and semantics of modal propositional and predicate logic
  - different proof methods: axiomatic calculi and tableaux methods
  - the role of models in logic
  - soundness and completeness of proof methods
  - epistemic logic, deontic logic, temporal logic, provability logic
  - the interpretation of modals and conditionals in natural language
  - and much more
- You must have taken Logic 1 or something equivalent.
- Logic 1 was easy. This course will be harder.

# Course info

### Website, Readings, Exercises

- The website for this course is *www.wolfgangschwarz.net/logic2*.
- Each week I will post extensive lecture notes with exercises.
- You are strongly advised to read the notes and attempt to answer the exercises.

# Course info

## Tutorials

- Tutorials run for two hours.
- In the first hour, we go through the answers to selected exercises.
- The second hour is an open Q&A session, often with more answers to exercises.
- Only the first hour is compulsory.

# Course info

## Assessment

- 20% First take-home test (probably 28/02–03/03)
- 30% Second take-home test (probably 28/03–31/03)
- 50% Final take-home test/exam (probably sometime in May)

# What is modal logic?

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In propositional logic, we can formalize arguments like this:

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It is either raining or snowing.
It is not snowing.

---

It is raining.

# What is modal logic?

In propositional logic, we can formalize arguments like this:

| | |
|---|---|
| It is either raining or snowing. | $r \vee s$ |
| It is not snowing. | $\neg s$ |
| It is raining. | $r$ |

# What is modal logic?

In propositional logic, we can formalize arguments like this:

| | |
|---|---|
| It is either raining or snowing. | $r \lor s$ |
| It is not snowing. | $\neg s$ |
| It is raining. | $r$ |

Remember: $\neg$, $\land$, $\lor$, $\rightarrow$, $\leftrightarrow$

# What is modal logic?

In propositional logic, we can formalize arguments like this:

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| It is either raining or snowing. | $r \vee s$ |
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| It is raining. | $r$ |

Remember: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

We can show that the argument is valid.

# What is modal logic?

We can also formalize this argument:

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We can also formalize this argument:

All birds have feathers.
All penguins are birds.

---

All penguins have feathers.

# What is modal logic?

We can also formalize this argument:

| | |
|---|---|
| All birds have feathers. | $p$ |
| All penguins are birds. | $q$ |
| All penguins have feathers. | $r$ |

# What is modal logic?

We can also formalize this argument:

| | |
|---|---|
| All birds have feathers. | $p$ |
| All penguins are birds. | $q$ |
| All penguins have feathers. | $r$ |

But in the language of propositional logic we can't bring out why the argument is valid.

# What is modal logic?

We need to **extend our formal language**.

All birds have feathers.
All penguins are birds.

---

All penguins have feathers.

# What is modal logic?

We need to **extend our formal language**.

| | |
|---|---|
| All birds have feathers. | $\forall x(Bx \rightarrow Fx)$ |
| All penguins are birds. | $\forall x(Px \rightarrow Bx)$ |
| All penguins have feathers. | $\forall x(Px \rightarrow Fx)$ |

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Remember: $\forall$, $\exists$

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Remember: $\forall$, $\exists$

We can now show that the argument is valid.

# What is modal logic?

Another argument:

# What is modal logic?

Another argument:

It is possible that it is raining.
It is certain that we will get wet if it is raining.

---

It is possible that we will get wet.

# What is modal logic?

Another argument:

| | | |
|---|---|---|
| It is possible that it is raining. | $p$ | $p$ |
| It is certain that we will get wet if it is raining. | $q$ | $r \to q$ |
| It is possible that we will get wet. | $r$ | $s$ |

# What is modal logic?

Every argument of this form is plausibly valid:

It is possible that *A*.
It is certain that *B* if *A*.

---

It is possible that *B*.

# What is modal logic?

We need to **extend our formal language**.

| | |
|---|---|
| It is possible that it is raining. | $\Diamond r$ |
| It is certain that we will get wet if it is raining. | $\Box(r \to w)$ |
| It is possible that we will get wet. | $\Diamond w$ |

# What is modal logic?

We need to **extend our formal language**.

| | |
| --- | --- |
| It is possible that it is raining. | $\Diamond r$ |
| It is certain that we will get wet if it is raining. | $\Box(r \rightarrow w)$ |
| It is possible that we will get wet. | $\Diamond w$ |

$\Diamond$ translates 'it is possible that ...'.

$\Box$ translates 'it is certain that ...'.

## What is modal logic?

Another argument:

# What is modal logic?

Another argument:

You are allowed to come in.
You must take off your shoes if you come in.

---

You are allowed to take off your shoes.

# What is modal logic?

Another argument:

| | |
|---|---|
| You are allowed to come in. | $\mathrm{P}\, c$ |
| You must take off your shoes if you come in. | $\mathrm{O}(c \to s)$ |
| You are allowed to take off your shoes. | $\mathrm{P}\, s$ |

P translates ‘it is permitted that ...’.

O translates ‘it is obligatory that ...’

# What is modal logic?

Modal logic is used to formalise reasoning about

- possibility and necessity
- permission and obligation
- knowledge and ignorance
- past, present, and future
- provability in mathematical theories
- the processing of computer programs
- and many other topics

# What is modal logic?

Modal logic is used to formalise reasoning about

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- and many other topics

The box $\Box$ *often* means ‘it is necessary that’ and the diamond $\Diamond$ ‘it is possible that’, but these expressions in turn can mean different things.

# Translating from English

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- John can’t leave the room.

# Translating from English

- John can't leave the room.
- $\neg \lozenge p$ ($p$: John leaves the room)

# Translating from English

- John can't leave the room.
- $\neg\Diamond p$ ($p$: John leaves the room)
- It is not possible that John leaves the room.

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- John mustn't leave the room.

# Translating from English

- John can't leave the room.
- $\neg\Diamond p$ ($p$: John leaves the room)
- It is not possible that John leaves the room.

- John mustn't leave the room.
- $\Box\neg p$ or O $\neg p$

# Translating from English

- John can't leave the room.
- $\neg\Diamond p$ ($p$: John leaves the room)
- It is not possible that John leaves the room.

- John mustn't leave the room.
- $\Box\neg p$ or O $\neg p$
- It is necessary/obligatory that John does not leave the room.

# Translating from English

- Going to lectures is no guarantee that you'll do well in the exam.

# Translating from English

- Going to lectures is no guarantee that you'll do well in the exam.
- $\neg\Box(g \rightarrow w)$ ($g$: You go to the lectures, $w$: You do well in the exam)

# Translating from English

- Going to lectures is no guarantee that you'll do well in the exam.
- $\neg\Box(g \rightarrow w)$ ($g$: You go to the lectures, $w$: You do well in the exam)
- It is not necessary/certain that if you go to lectures you will do well in the exam.

# Translating from English

- If the lights are on, Ada might be in her office.

# Translating from English

- If the lights are on, Ada might be in her office.
- $\Diamond(l \land o)$ ($l$: The lights are on, $o$: Ada is in her office)

# Translating from English

- If the lights are on, Ada might be in her office.
- $\Diamond(l \wedge o)$ ($l$: The lights are on, $o$: Ada is in her office)
- It is possible that the lights are on and Ada is in her office.