

Logic 2: Modal Logic

Lecture 9

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A model of knowledge

A model of knowledge

In epistemic logic, the box represents knowledge.

Possible-worlds analysis of knowledge

S knows that P iff P is true at all worlds compatible with S 's knowledge.

In epistemic Kripke models, wRv means v is compatible with the agent's knowledge at w .

A model of knowledge

More knowledge = fewer open possibilities

The duke has been murdered. There are four suspects: the gardener, the butler, the cook, and the maid.



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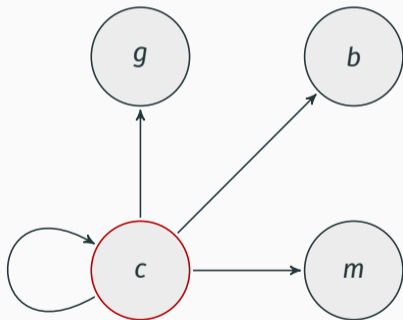
The gardener has an alibi.



A model of knowledge

More knowledge = fewer accessible worlds

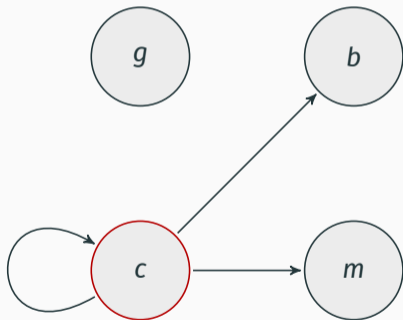
The cook has murdered the duke. The detective investigates.



A model of knowledge

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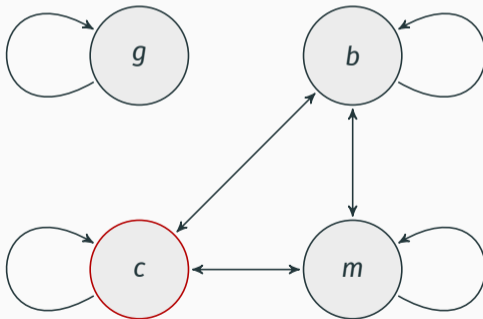
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A model of knowledge

The cook has murdered the duke. The detective investigates.

The gardener has an alibi.



A model of knowledge

In epistemic logic, we usually write the box as 'K'.

I write the diamond as 'M'.

Kripke semantics

$M, w \models \Box A$ iff $M, v \models A$ for all v such that wRv .

$M, w \models \Diamond A$ iff $M, v \models A$ for some v such that wRv .

Kripke semantics

$M, w \models KA$ iff $M, v \models A$ for all v such that wRv .

$M, w \models MA$ iff $M, v \models A$ for some v such that wRv .

Logical Omniscience

A consequence of Kripke semantics:

- Knowledge is **closed under known consequence**: $KA, K(A \rightarrow B) \models KB$.
- Knowledge is **closed under logical consequence**: If $A \models B$ then $\models KA \rightarrow KB$.

This seems wrong.

Response 1: Our semantics is only adequate for ideal agents.

Response 2: We are modelling (a tidied-up concept of) **implicit knowledge**.

Fred Dretske's (1970) argument against (K):

1. I know that I have hands. Kp
2. I know that if I have hands then I'm not a brain in a vat. $K(p \rightarrow \neg q)$
3. I do not know that I'm not a brain in a vat. $\neg K \neg q$

$K(A \rightarrow B), KA \models KB$

$\models K(A \rightarrow B) \rightarrow (KA \rightarrow KB)$

Epistemic Accessibility

A world v is epistemically accessible for an agent at w (wRv) iff

- the agent's knowledge at w is compatible with the hypothesis that v is the actual world;
- v might be the actual world, for all the agent knows;
- whatever the agent knows at w is true at v .

Can we be more informative?

Proposal 0

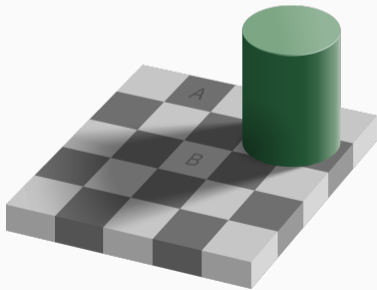
A world v is epistemically accessible for an agent at w (wRv) iff the agent's evidence at w is compatible with v .

An agent's evidence is what her senses tell her.

Proposal 1

A world v is epistemically accessible for an agent at w (wRv) iff the agent's evidence at w is compatible with v .

An agent's evidence is what her senses and memory tell her.



My senses tell me that square A is darker than square B.

But square A is not darker.

And I don't believe that square A is darker.

$Kp \rightarrow p$ would become invalid.

$Kp \rightarrow Bp$ would become invalid.

Proposal 2

A world v is epistemically accessible for an agent at w (wRv) iff the agent's sense experiences and memory at v are the same as at w .

- Is R reflexive? (For all w , wRw)
- Is R transitive? (If wRv and vRu then wRu)
- Is R symmetric? (If wRv then vRw)

Yes. R is an equivalence relation.

Proposal 2

A world v is epistemically accessible for an agent at w (wRv) iff the agent's sense experiences and memory at v are the same as at w .

We get an S5 logic.

$$(K) \quad K(A \rightarrow B) \rightarrow (KA \rightarrow KB)$$

$$(T) \quad KA \rightarrow A$$

$$(4) \quad KA \rightarrow KKA$$

$$(B) \quad A \rightarrow KMA$$

$$(5) \quad MA \rightarrow KMA$$

Proposal 2

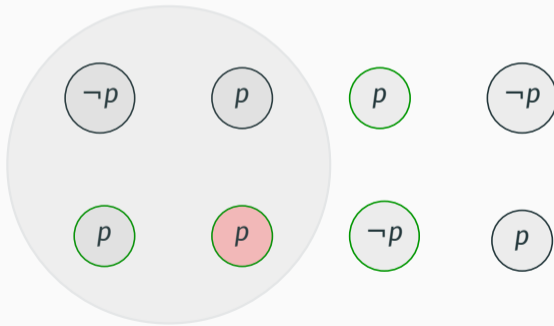
A world v is epistemically accessible for an agent at w (wRv) iff the agent's sense experiences and memory at v are the same as at w .

We also get scepticism about the external world.



Proposal 3 (Lewis 1996)

A world v is epistemically accessible for an agent at w (wRv) iff the agent's sense experiences and memory at v are the same as at w and v is not properly ignored



Formal properties of epistemic accessibility

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Almost everyone wants the logic of knowledge to validate

$$(T) \quad KA \rightarrow A$$

So R should be reflexive. We then automatically get

$$(D) \quad KA \rightarrow MA$$

Formal properties of epistemic accessibility

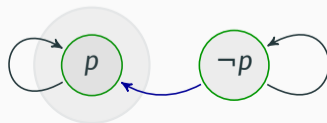
Should R be symmetric? Do we want (B) to come out valid?

$$(B) A \rightarrow KMA$$

Suppose you falsely believe $\neg p$.

- p .
- You believe that you know $\neg p$.
- You don't believe that you don't know $\neg p$.
- You don't know that you don't know $\neg p$.
- $\neg K \neg K \neg p$.
- $\neg KM p$.

Also, this would lead to skepticism.



Positive Introspection:

$$(4) KA \rightarrow KKA$$

Negative Introspection:

$$(5) MA \rightarrow KMA$$

(5) corresponds to euclidity. Euclidity and reflexivity entail symmetry. So philosophers mostly reject (5).

(4) corresponds to transitivity. It is controversial.

Feedback

Midsemester Feedback Logic 2

