Logic 2: Modal Logic

Lecture 12

Wolfgang Schwarz

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University of Edinburgh

Review

Review

Kripke semantics for obligation and permission

$M, w \models \mathrm{O}A$ iff $M, v \models A$ for all v with wRv.
$M, w \models \mathrm{P}A$ iff $M, v \models A$ for some v with wRv.

What is R?

- wRv iff all norms are respected at v.
- wRv iff all norms at w are respected at v.

Assuming seriality, the resulting deontic logic is either KD45 or D.

Review

Kripke semantics for deontic logic faces many problems.

- Response 1: replace Kripke semantics by a different semantics.
- Response 2: explain away the problems.
    - Appeal to pragmatics.
    - Redefine the accessibility relation.
    - Offer a new translation for deontic conditionals.

Neighbourhood Semantics

Neighbourhood Semantics

Let's drop the assumption that $\Box$ and $\Diamond$ are quantifiers over worlds.

For every sentence A there is a set of worlds $[A]$ at which A is true.

We assume that the box and the diamond express properties of these sets.

$\Box A$ is true iff $[A]$ is required/necessary/known/etc. $\Box A$ is true at w iff $[A]$ is required/necessary/known/etc at w. $\Box A$ is true at w iff $[A]$ is in the "neighbourhood" of w.

Neighbourhood Semantics

A **neighbourhood model** consists of

- a non-empty set W,
- a function N that assigns to each member of W a set of subsets of W, and
- a function V that assigns to each sentence letter of $\mathfrak{L}_M$ a subset of W.

Neighbourhood semantics

$$M, w \models \Box A \text{ iff } [A]^M \text{ is in } N(w).$$

$$M, w \models \Diamond A \text{ iff } [\neg A]^M \text{ is not in } N(w).$$

Neighbourhood Semantics

The set of sentences that are true at all worlds in all neighbourhood models is called **system E.** It is axiomatized by

(**Dual**) $\neg\Box A \leftrightarrow \Diamond\neg A$

(**CPL**) Any truth-functional consequence of sentences in the system is in the system.

(**RE**) If $A \leftrightarrow B$ is in the system then so is $\Box A \leftrightarrow \Box B$.

Neighbourhood Semantics

(K), (T), (D), (B), (4), (5), and (G) are not valid in neighbourhood semantics. Nor are

- $\Box p \wedge \Box q \rightarrow \Box(p \wedge q)$
- $\Box(p \vee \neg p)$

Pragmatics

Ross's Paradox

(3) You must either mail the letter or burn it.

$\rightsquigarrow$ (3 a) You are permitted to mail the letter.

$\rightsquigarrow$ (3 b) You are permitted to burn the letter.

Pragmatics

(1) Amy is at the pub or in the library.

⇝ (1a) Amy might be at the pub.

⇝ (1b) Amy might be in the library.

This is not an entailment, but an **implicature**.

- It would (normally) be uncooperative to assert (1) unless (1a) and (1b) were true.
- So we take an assertion of (1) to indicate the truth of (1a) and (1b).

Pragmatics

(2) All students got a First or a Second.

$\rightsquigarrow$ (2 a) Some students got a First.

$\rightsquigarrow$ (2 b) Some students got a Second.

Ross’s Paradox

(3) You must either mail the letter or burn it.

$\rightsquigarrow$ (3 a) You are permitted to mail the letter.

$\rightsquigarrow$ (3 b) You are permitted to burn the letter.

The Paradox of Free Choice

(4) You may have beer or wine.

$\rightsquigarrow$ (4 a) You may have beer.

$\rightsquigarrow$ (4 b) You may have wine.

A Sign that this is an implicature:

- 'You may not have beer' does not imply 'You may not have beer or wine'.

Redefining Accessibility

The Bank Robber Paradox

Mary robbed a bank.

1. Mary ought to go to jail. $\mathrm{O}\, j$
2. Mary ought to not have robbed the bank. $\mathrm{O}\, \neg r$
3. But not: $\mathrm{O}(j \wedge \neg r)$.

In an ideal world, Mary didn't rob the bank and doesn't go to jail.

Assuming wRv iff v is ideal (relative to w), then $\mathrm{O}\, j$ is false.

Redefining Accessibility

Why is it true that Mary ought to go to jail?

Because she has robbed the bank, and this is settled.

Among the open worlds, the best worlds are worlds where Mary goes to jail.

Redefining Accessibility

New definition of accessibility:

> wRv iff v is one of the best worlds (by w's norms) among those that are open at w.

Obligations depend not just on the norms, but also on the circumstances.

- Circumstance: Mary has robbed a bank.
- Norm: Anyone who has robbed a bank must go to jail.
- So: Mary must go to jail.

Redefining Accessibility

New definition of accessibility:

> *wRv* iff *v* is one of the best worlds (by *w*'s norms) among those that are open at *w*.

Obligations depend not just on the norms, but also on the circumstances.

- Circumstance: You walk past a drowning baby.
- Norm: Don't let babies drown.
- So: You must rescue the baby.

Redefining Accessibility

Now we have a problem with “Mary ought not to have robbed the bank”.

Here we consider worlds that aren’t open.

Revised new definition of accessibility:

> wRv iff v is one of the best worlds (by w’s norms) among those that are circumstantially possible at w.

What is circumstantially possible depends on conversational context.

Redefining Accessibility

The Bank Robber Paradox

1. Mary ought to go to jail. $\mathrm{O}\, j$
2. Mary ought to not have robbed the bank. $\mathrm{O}\, \neg r$
3. But not: $\mathrm{O}(j \wedge \neg r)$.

Different circumstances are relevant in 1 and 2.

If we don't equivocate, either $\mathrm{O}\, j$ or $\mathrm{O}\, \neg r$ is false.

Redefining Accessibility

The Samaritan Paradox

1. Jones ought to help the injured Smith.
2. That Jones helps the injured Smith entails that Smith has been injured.
3. But not: Smith ought to have been injured.

Different circumstances are (pragmatically) held fixed in 1 and 3.

Professor Procrastinate

1. Professor Procrastinate ought not to accept the review.
2. Professor Procrastinate ought to accept and complete the review.

Different circumstances are held fixed in 1 and 2.

Deontic conditionals

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(*) If John kills his mother, he should kill her gently.

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Intuitively: (*) says that at the best worlds among those at which John kills his mother, he kills her gently.

This cannot be expressed in $\mathfrak{L}_M$.

But we can add a binary obligation operator $\mathrm{O}(\cdot/\cdot)$ to express it: $\mathrm{O}(q/p)$.

> $\mathrm{O}(q)$ is true iff q is true at the best worlds at which relevant circumstances obtain.
>
> $\mathrm{O}(q/p)$ is true iff q is true at the best worlds at which relevant circumstances obtain and p is the case.