

# Logic 2: Modal Logic

## Lecture 14

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# Review

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## Temporal operators:

- F: It will (at some point) be the case that
- G: It is always going to be the case that
- P: It was (at some point) the case that
- H: It has always been the case that

### Temporal Model

A **temporal model** consists of

- a non-empty set  $T$  (of “times”),
- a binary relation  $<$  on  $T$  (the **precedence relation**),
- a function  $V$  that assigns to each sentence letter a subset of  $T$ .

## Standard Temporal Semantics

- (g)  $M, t \models FA$  iff  $M, s \models A$  for some  $s$  such that  $t < s$ .
- (h)  $M, t \models GA$  iff  $M, s \models A$  for all  $s$  such that  $t < s$ .
- (i)  $M, t \models PA$  iff  $M, s \models A$  for some  $s$  such that  $s < t$ .
- (j)  $M, t \models HA$  iff  $M, s \models A$  for all  $s$  such that  $s < t$ .

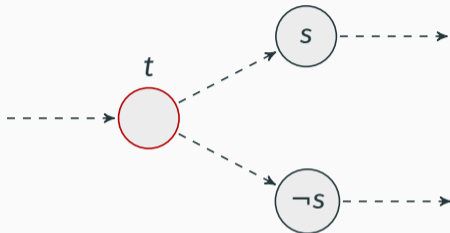
The logic of time depends on formal properties of the precedence relation.

- We always assume that  $>$  is transitive. This renders  $GA \rightarrow GGA$  valid.
- We might assume that  $>$  is asymmetric and irreflexive. This doesn't affect the logic.
- We might assume that  $>$  is discrete. This would render  $(A \wedge HA) \rightarrow FHA$  valid.
- We might assume that  $>$  is left-linear. This would render  $FPA \rightarrow (FA \vee A \vee PA)$  valid.
- We might assume that  $>$  is right-linear. This would render  $PFA \rightarrow (PA \vee A \vee FA)$  valid.

## Branching time

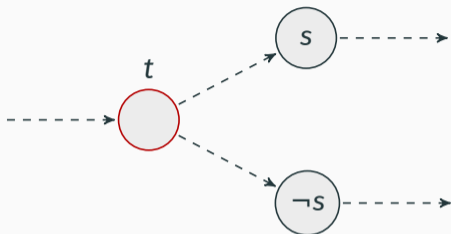
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## Branching time



Is  $Fs$  true at  $t$ ? Yes.

Intuition: 'There will be a sea battle' is **not true** at  $t$ .



### Standard semantics:

$M, t \models FA$  iff  $M, t' \models A$  for some  $t'$  such that  $t < t'$   
iff  $A$  is true at **some** future point on **some** history through  $t$ .

### “Peircean” semantics (CTL):

$M, t \models FA$  iff  $A$  is true at **some** future point on **every** history through  $t$ .

Is  $Fs$  true at  $t$  in Peircean semantics? No.

### Standard semantics:

$M, t \models FA$  iff  $A$  is true at **some** future point on **some** history through  $t$ .

### “Peircean” semantics (CTL):

$M, t \models FA$  iff  $A$  is true at **some** future point on **every** history through  $t$ .

We can factor out the quantification over histories.

$\Box A$ : On every history (through the present point) ...

$\Diamond A$ : On some history (through the present point) ...

$\Diamond FA$ :  $A$  is true at **some future** point on **some** history through  $t$ .

$\Box FA$ :  $A$  is true at **some future** point on **every** history through  $t$ .

## Branching time

$\Box A$ : On every history (through the present point) ...

$\Diamond A$ : On some history (through the present point) ...

$FA$ : At some point in the future ...

How does this language work?

$M, t \models \Diamond A$  iff ?

$M, t \models FA$  iff ?

**“Ockhamist” semantics (CTL\*):**

$M, h, t \models \Diamond A$  iff  $M, h', t \models A$  for some history  $h'$  that contains  $t$ .

$M, h, t \models FA$  iff  $M, h, t' \models A$  for some  $t'$  with  $t < t'$ .

### “Ockhamist” semantics (CTL\*)

$M, h, t \models \Diamond A$  iff  $M, h', t \models A$  for some history  $h'$  that contains  $t$ .

$M, h, t \models FA$  iff  $M, h, t' \models A$  for some  $t'$  with  $t < t'$ .

Truth is defined relative to three **parameters**:  $M, h, t$ .

Only  $M$  and  $t$  represent a scenario and an interpretation.

Ockhamism doesn't tell us which sentences are true in a given scenario under a given interpretation.

So it doesn't tell us which sentences are true in all scenarios under all interpretations.

### “Ockhamist” semantics (CTL\*)

$M, h, t \models \Diamond A$  iff  $M, h', t \models A$  for some history  $h'$  that contains  $t$ .

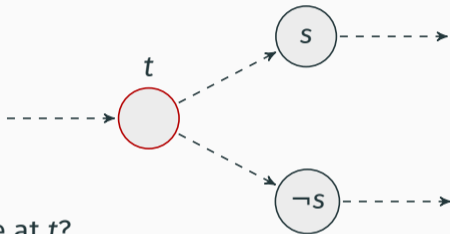
$M, h, t \models FA$  iff  $M, h, t' \models A$  for some  $t'$  with  $t < t'$ .

### Supervaluationism:

$M, t \models A$  iff  $M, h, t \models A$  for **every** history  $h$  through  $t$ .

## Branching time

$M, t \models A$  iff  $M, h, t \models A$  for *every* history  $h$  through  $t$ .



Which of these are true at  $t$ ?

- $\diamond F_s$
- $\square F_s$
- $F_s$
- $\neg F_s$
- $F_s \vee \neg F_s$

Supervaluationist Ockhamism determines a **three-valued logic**. A sentence can be

- true
- false
- neither

The truth-value of a truth-functionally complex sentence at a scenario is not determined by the truth-value of the parts:

- $Fs$  and  $\neg Fs$  are neither true nor false,  $Fs \vee \neg Fs$  is true.
- $Fs$  and  $Fs$  are neither true nor false,  $Fs \vee Fs$  is neither true nor false.

## Branching time

In other three-valued logics, the truth-value of truth-functionally complex sentences is determined by the truth-values of the parts:

A	B	$A \vee B$
1	1	1
1	N	1
1	0	1
N	1	1
N	N	N
N	0	N
0	1	1
0	N	N
0	0	0