Logic 2: Modal Logic

Lecture 15

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Conditionals in natural language

Conditionals in natural language

- If Russia invades Estonia, NATO will attack Russia.
- If we don’t reduce greenhouse gases, the climate might get out of control.
- You will be faster if you take a taxi.
- If Heisenberg hadn’t undermined the Nazi’s nuclear weapons programme, Germany would have won the war.
- If Jones hadn’t untied the rope, Smith would not have fallen.

Conditionals in natural language

Indicative:

- If Oswald did not kill Kennedy, someone else did.

Subjunctive/counterfactual:

- If Oswald had not killed Kennedy, someone else would have.

Material conditionals

Material conditionals

$A \rightarrow B$ is a "material conditional": it is true iff A is false or B is true.

A	B	$A \rightarrow B$
1	1	1
1	0	0
0	1	1
0	0	1

The material analysis: 'If A then B' is a material conditional.

A	B	if A then B
1	1	1
1	0	0
0	1	1
0	0	1

Material conditionals

A quick argument for the material analysis

1. ‘If *A* then *B*’ entails ‘not *A* or *B*’.
2. ‘*A* or *B*’ entails ‘if not *A* then *B*’.
3. ‘Not *A* or *B*’ entails ‘if not not *A* then *B*’.
4. ‘Not *A* or *B*’ entails ‘if *A* then *B*’.

Material conditionals

Another quick argument for the material analysis

- ‘If n is a prime number greater than 2 then n is odd.’
- ‘For any number n, if n is a prime number greater than 2 then n is odd.’
- ‘If 1 is a prime number greater than 2 then 1 is odd.’
- ‘If 2 is a prime number greater than 2 then 2 is odd.’
- ‘If 3 is a prime number greater than 2 then 3 is odd.’
- ...

Another argument, due to Alan Gibbard (1981)

Modus Ponens: ‘If *A* then *B*’ and *A* entail *B*.

Import-Export: ‘If *A* then if *B* then *C*’ is equivalent to ‘if *A* and *B* then *C*’.

(1) ‘If not *A* or *B* then if *A* then *B*.’

By Import-Export, (1) is equivalent to the tautology

(2) ‘If [not *A* or *B*] and *A* then *B*.’

So (1) is a logical truth.

By Modus Ponens, (1) and ‘not *A* or *B*’ entail ‘if *A* then *B*’.

So ‘not *A* or *B*’ entails ‘if *A* then *B*’.

Also, ‘if *A* then *B*’ entails ‘not *A* or *B*’.

The logic of material conditionals

		$A \rightarrow B$
Modus Ponens	if A then B, A ∴ B	valid
Conditional Proof	A entails B ∴ if A then B	valid
Or-to-If	A or B ∴ if A then B	valid
Import-Export	if A then if B then C ∴.∴ if A and B then C	valid
Contraposition	if A then B ∴ if not B then not A	valid
Transitivity	if A then B, if B then C ∴ if A then C	valid
SDA	if A or B then C ∴ if A then C and if B then C	valid
Antec. Strength.	if A then C ∴ if A and B then C	valid
False Antec.	not A ∴ if A then B	valid
True Cons.	B ∴ if A then B	valid

Material conditionals

1. *True Cons.* $B \therefore$ if A then B
   The lecture ends at 1 pm. Therefore: If the building collapses at 12.45 then the lecture ends at 1 pm.
2. *False Antec.* not $A \therefore$ if A then B
   It is not the case that if it will rain tomorrow then the Moon will fall onto the Earth. Therefore: It will rain tomorrow.
3. *Antec. Strength.* if A then $C \therefore$ if A and B then C
   If you add sugar to your coffee, it will taste good. Therefore: If you add sugar and vinegar to your coffee, it will taste good.
4. *Contraposition* if A then $B \therefore$ if not B then not A
   If our opponents are cheating, we will never find out. Therefore: If we will find out that our opponents are cheating, then they aren't cheating.

Strict conditionals

Strict conditionals

Russell and Whitehead, *Principia Mathematica* (1913):

> *...if p and $\neg p \vee q$ are both true, then q is true. In this sense, the proposition $\neg p \vee q$ will be quoted as stating that p implies q. (p.7)*

So 'the building collapses at 12.45' implies 'the lecture ends at 1 pm'.

C.I. Lewis (1918):

- $\neg p \vee q$ is not a good formalization of 'p implies q'.
- A better one is $\Box(p \rightarrow q)$.

Some have argued that $\Box(p \rightarrow q)$ is also a good formalization of 'if p then q'.

Strict conditionals

Define $A \strictif B$ as $\Box(A \rightarrow B)$.

Kripke semantics for $\strictif$

If $M = \langle W, R, V \rangle$ is a Kripke model, then

$M, w \models A \strictif B$ iff for all v such that wRv, $M, v \not\models A$ or $M, v \models B$.

What is R?

Strict conditionals

- If Oswald did not kill Kennedy then someone else did.
- $\Box(p \rightarrow q)$

Hypothesis: wRv iff v is compatible with what is known at w.

Modus Ponens is valid because epistemic accessibility is reflexive.

- Suppose $\Box(A \rightarrow B)$ and A.
- $\Box(A \rightarrow B)$ entails $A \rightarrow B$.
- $A \rightarrow B$ and A entail B.
- So B.

Strict conditionals

		$A \to B$	$A \strictif B$
Modus Ponens	if *A* then *B*, *A* ∴ *B*	valid	valid
Conditional Proof	*A* entails *B* ∴ if *A* then *B*	valid	valid
Or-to-If	$A \vee B$ ∴ if *A* then *B*	valid	invalid
Import-Export	if *A* then if *B* then *C* ∴∴ if *A* and *B* then *C*	valid	invalid
Contraposition	if *A* then *B* ∴ if not *B* then not *A*	valid	valid
Transitivity	if *A* then *B*, if *B* then *C* ∴ if *A* then *C*	valid	valid
SDA	if *A* or *B* then *C* ∴ if *A* then *C* and if *B* then *C*	valid	valid
Antec. Strength.	if *A* then *C* ∴ if *A* and *B* then *C*	valid	valid
False Antec.	not *A* ∴ if *A* then *B*	valid	invalid
True Cons.	*B* ∴ if *A* then *B*	valid	invalid

Strict conditionals

Problems:

- $A \strictif B \models \neg B \strictif \neg A$
  If our opponents are cheating, we will never find out. Therefore: If we will find out that our opponents are cheating, then they aren't cheating.
- $A \strictif B \models (A \wedge C) \strictif \neg B$
  If you add sugar to your coffee, it will taste good. Therefore: If you add sugar and vinegar to your coffee, it will taste good.
- $A \strictif B, B \strictif C \models A \strictif C$.
  If I quit my job, I won't be able to pay rent. If I win a million, I'll quit my job. Therefore: if I win a million, I won't be able to pay rent.

Strict conditionals

Possible response:

The accessibility relation depends on conversational context.

- 'If you add sugar to your coffee, it will taste good.'
  - Here worlds where you add sugar and vinegar to your coffee are ignored/inaccessible.
- 'If you add sugar and vinegar to your coffee, it will taste good.'
  - Now these worlds are no longer ignored/inaccessible.

Another problem

Why are we often unsure about conditionals?

- I'm not sure whether NATO will attack Russia if Russia invades Estonia.

This is not because I'm unsure about what I know.