

# Logic 2: Modal Logic

## Lecture 9

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# Kripke models for knowledge

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In epistemic logic, the box represents knowledge.

### Possible-worlds analysis of knowledge

$S$  knows that  $P$  iff  $P$  is true at all worlds compatible with  $S$ 's knowledge.

In epistemic Kripke models,  $wRv$  means  $v$  is compatible with the agent's knowledge at  $w$ .

# Kripke models for knowledge

**More knowledge = fewer open possibilities**

The duke has been murdered. There are four suspects: the gardener, the butler, the cook, and the maid.



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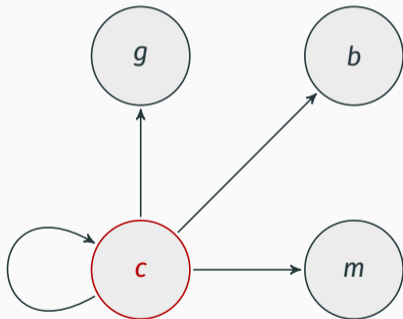
The gardener has an alibi.



## Kripke models for knowledge

**More knowledge = fewer accessible worlds**

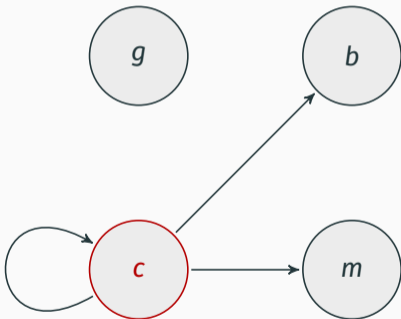
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## Kripke models for knowledge

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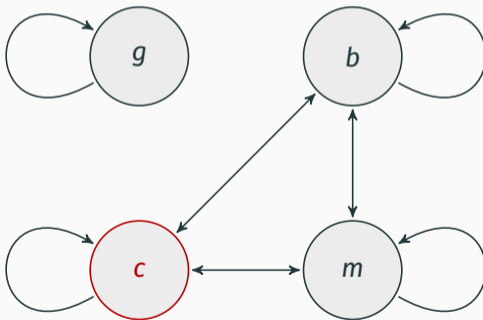
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## Kripke models for knowledge

The cook has murdered the duke. The detective investigates.

The gardener has an alibi.





### Kripke semantics

$M, w \models \Box A$  iff  $M, v \models A$  for all  $v$  such that  $wRv$ .

$M, w \models \Diamond A$  iff  $M, v \models A$  for some  $v$  such that  $wRv$ .

In epistemic logic, we usually write the box as 'K'.

I write the diamond as 'M'.

### Kripke semantics

$M, w \models KA$  iff  $M, v \models A$  for all  $v$  such that  $wRv$ .

$M, w \models MA$  iff  $M, v \models A$  for some  $v$  such that  $wRv$ .

# Logical Omniscience

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A consequence of Kripke semantics:

- Knowledge is **closed under known consequence**:  $KA, K(A \rightarrow B) \models KB$ .
- Knowledge is **closed under logical consequence**: If  $A \models B$  then  $\models KA \rightarrow KB$ .

This seems wrong.

Response 1: Kripke semantics is only adequate for ideal agents.

Response 2: We are modelling **implicit knowledge**.

### One motivation for studying implicit/ideal knowledge:

Intuitively, one can know  $p$  and fail to know  $q$  even if  $p$  and  $q$  are logically equivalent.

So the objects of knowledge can't be sets of possible worlds.

Suppose they are sentences.

*(\* Nobody knows the fourth sentence on slide 9.*

If (\*) is false, then somebody does know (\*); that's impossible.

So (\*) is true. I.e., nobody knows (\*).

???

## Fred Dretske's (1970) case against K:

1. I know that I have hands.  $Kp$
2. I know that if I have hands then I'm not a brain in a vat.  $K(p \rightarrow \neg q)$
3. I do not know that I'm not a brain in a vat.  $\neg K \neg q$

$$\models K(A \rightarrow B) \rightarrow (KA \rightarrow KB)$$

$$K(A \rightarrow B), KA \models KB$$

# Epistemic Accessibility

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A world  $v$  is epistemically possible for an agent at  $w$  ( $wRv$ ) iff

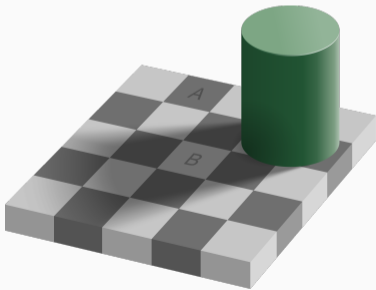
- the agent's knowledge at  $w$  is compatible with the hypothesis that  $v$  is the actual world;
- $v$  might be the actual world, for all the agent knows;
- whatever the agent knows at  $w$  is true at  $v$ ;

Can we be more informative?

## Proposal 1

A world  $v$  is epistemically possible for an agent at  $w$  ( $wRv$ ) iff the agent's evidence at  $w$  is compatible with  $v$ .

An agent's evidence is what her senses and memory tell her.



My senses tell me that square A is darker than square B.

But I know they are the same shade of grey.

$KA \rightarrow A$  would become invalid.

$KA \rightarrow BA$  would become invalid.



### Proposal 2

A world  $v$  is epistemically possible for an agent at  $w$  ( $wRv$ ) iff the agent's sense experiences and memory at  $v$  are the same as at  $w$ .

- Is  $R$  reflexive? (For all  $w$ ,  $wRw$ )
- Is  $R$  transitive? (If  $wRv$  and  $vRu$  then  $wRu$ )
- Is  $R$  symmetric? (If  $wRv$  then  $vRw$ )

Yes.  $R$  is an equivalence relation.

## Proposal 2

A world  $v$  is epistemically possible for an agent at  $w$  ( $wRv$ ) iff the agent's sense experiences and memory at  $v$  are the same as at  $w$ .

We get an S5 logic.

$$(K) \quad K(A \rightarrow B) \rightarrow (KA \rightarrow KB)$$

$$(T) \quad KA \rightarrow A$$

$$(4) \quad KA \rightarrow KKA$$

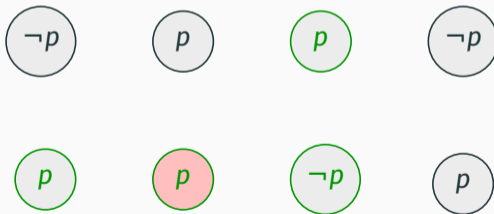
$$(B) \quad A \rightarrow KMA$$

$$(5) \quad MA \rightarrow KMA$$

## Proposal 2

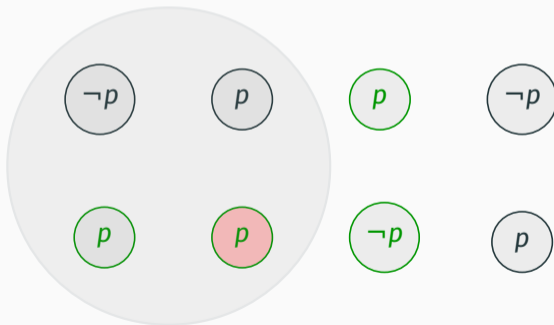
A world  $v$  is epistemically possible for an agent at  $w$  ( $wRv$ ) iff the agent's sense experiences and memory at  $v$  are the same as at  $w$ .

We also get scepticism about the external world.



## Proposal 3 (Lewis 1996)

A world  $v$  is epistemically possible for an agent at  $w$  ( $wRv$ ) iff the agent's sense experiences and memory at  $v$  are the same as at  $w$  and  $v$  is not properly ignored



**Reflexivity, Seriality, Symmetry,  
Transitivity, Euclidity**

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Almost everyone wants the logic of knowledge to validate

$$(T) \quad KA \rightarrow A$$

So  $R$  should be reflexive. We then automatically get

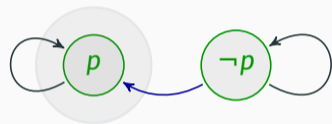
$$(D) \quad KA \rightarrow MA$$

Should  $R$  be symmetric? Do we want **B** to come out valid?

(B)  $A \rightarrow KMA$

This would lead to skepticism.

Also: suppose you have misleading evidence for  $\neg p$ .



- $p$  is true.
- You believe that you know  $\neg p$ .
- You don't believe that you don't know  $\neg p$ .
- So you don't know that you don't know  $\neg p$ .
- $\neg KMp$

### Positive Introspection:

$$(4) \quad KA \rightarrow KKA$$

### Negative Introspection:

$$(5) \quad MA \rightarrow KMA$$

5 corresponds to euclidity. Euclidity and reflexivity entail symmetry. So philosophers mostly reject 5.

4 corresponds to transitivity. It is controversial.