Normal deontic logics
Deontic logic formalizes reasoning about norms.

- Obligation
- Permission
- Prohibition
- Optionality
- Rights
- Duties
- Supererogation
- etc.
Normal deontic logics

We focus on two operators:

- O: It is obligatory that ...
- P: It is allowed that ...

You must return the library book

⇒ It is obligatory that you return the library book
⇒ O\ p
Simple absolutist Kripke semantics

Call a world **ideal** if it contains no violations of any (relevant) norms.

O $A$ is true iff $A$ is true at all ideal worlds.

P $A$ is true iff $A$ is true at some ideal world.
Normal deontic logics

Simple absolutist Kripke semantics

\[ M, w \models O A \iff M, v \models A \text{ for all } v \text{ with } wRv. \]
\[ M, w \models P A \iff M, v \models A \text{ for some } v \text{ with } wRv. \]

Interpret \( wRv \) as: \( v \) is one of the ideal worlds.

- Is \( R \) reflexive (every world can see itself)?
- Is \( R \) serial (every world can see some world)?
- Is \( R \) transitive (if \( wRv \) and \( vRu \) then \( wRu \))?
- Is \( R \) symmetric (if \( wRv \) then \( vRw \))?
- Is \( R \) euclidean (if \( wRv \) and \( wRu \) then \( vRu \))?

Assuming seriality, we get the logic KD45.
Normal deontic logics

(Deontic) KD45 is axiomatized by the propositional schemas $\textbf{A1}$–$\textbf{A3}$ plus

\begin{align*}
(\textbf{K}) & \quad O(A \rightarrow B) \rightarrow (O A \rightarrow O B) \\
(\textbf{D}) & \quad OA \rightarrow PA \\
(4) & \quad OA \rightarrow O OA \\
(5) & \quad PA \rightarrow O PA
\end{align*}

and the rules $\textbf{MP}$ and $\textbf{Nec}$.
Normal deontic logics

Simple relativist Kripke semantics

\[ M, w \models O A \iff M, v \models A \text{ for all } v \text{ with } wRv. \]
\[ M, w \models P A \iff M, v \models A \text{ for some } v \text{ with } wRv. \]

Interpret \( wRv \) as: \( v \) is ideal relative to the norms of \( w \).

- Is \( R \) reflexive (every world can see itself)?
- Is \( R \) serial (every world can see some world)?
- Is \( R \) transitive (if \( wRv \) and \( vRu \) then \( wRu \))?  
- Is \( R \) symmetric (if \( wRv \) then \( vRw \))?  
- Is \( R \) euclidean (if \( wRv \) and \( wRu \) then \( vRu \))? 

Assuming seriality, we get the **standard deontic logic** \( D \).
Challenges to normal deontic logics
1. Obligatory tautologies

If $A$ is true at all worlds, then $OA$ is true at all worlds.

$\models_K O(p \lor \neg p)$

But are you obligated to either go to class or not go to class?
2. No scenarios without norms (Chellas 1980)

If there are no norms, then there are no obligations or permissions. It is not a logical truth that there are norms.

\[ \models_{\mathcal{K}} O(p \lor \neg p) \]
\[ \models_{\mathcal{K}} P(p \lor \neg p) \]
3. Conflicting obligations (Lemmon 1962)

You may be obligated to do \( p \) and obligated to do \( \neg p \), without being obligated to do everything.

\[ \models_K (O p \land O \neg p) \rightarrow O q. \]
4. The Samaritan Paradox (Prior 1958)

Smith has been robbed and injured.

- Jones ought to help the injured Smith.
- That Jones helps the injured Smith entails that Smith has been injured.

In Kripke semantics, if $O \alpha$ is true, and $\alpha$ entails $\beta$, then $O \beta$ is true.

- So: Smith ought to have been injured?!
5. The Knowledge Paradox (Aqvist 1967)

- Jones ought to know that there is a fire.
- That Jones knows that there is a fire entails that there is a fire.
- So there ought to be a fire?
Challenges to normal deontic logics

6. The Bank Robber Paradox

Mary robbed a bank.

- Mary ought to go to jail.
- Mary ought to not have robbed the bank.

\[ \models_k (O A \land O B) \rightarrow O(A \land B) \]

- So: it ought to be the case that Mary didn’t rob the bank and yet she goes to jail?
Challenges to normal deontic logics

7. Professor Procrastinate (Pargetter and Jackson 1986)

• Professor Procrastinate ought not to accept the review.
• Professor Procrastinate ought to accept and complete the review.

\[ \models K (O(A \land B) \rightarrow OA) \]
8. Ross’s Paradox (Ross 1943)

Intuitively,

- you must either mail or burn the letter

entails

- you are permitted to mail the letter, and
- you are permitted to burn the letter.

But $\vdash_k \Diamond m \rightarrow \Diamond (m \lor b)$

Intuitively,

\[
\begin{align*}
&\text{you may have beer or wine} \\
&\text{entails} \\
&\text{you are permitted to have beer, and} \\
&\text{you are permitted to have wine.}
\end{align*}
\]

But \( |=_{K} P \; b \rightarrow P(b \lor w). \)
10. The Gentle Murder Paradox (Forrester 1984)

- John ought to not buy meat.
- If he does buy meat, he should buy meat from sustainable sources.
- John does buy meat.

- \( \neg O p \)
- \( p \rightarrow O q \)
- \( p \)

By modus ponens, we can infer \( O q \). That seems wrong.

(Also, since \( q \) entails \( p \), we get \( O p \).)
10. The Gentle Murder Paradox (Forrester 1984)

- John ought to not buy meat.
- If he does buy meat, he should buy meat from sustainable sources.
- John does buy meat.

- $O \neg p$
- $O(p \rightarrow q)$
- $p$

Now we can no longer infer $O q$.

But $\models_k O \neg p \rightarrow O(p \rightarrow r)$.

So

- If John does buy meat, he should buy from factory farms?
Challenges to normal deontic logics

11. The Miners Puzzle (Kolodny and MacFarlane 2010)

- If the miners are in shaft A, we ought to block shaft A.
- If the miners are in shaft B, we ought to block shaft B.
- We ought to block neither shaft.

- \( S_A \lor S_B \)
- \( S_A \rightarrow O b_A \)
- \( S_B \rightarrow O b_B \)
- \( O(\neg b_A \land \neg b_B) \)

These are inconsistent in K.
Challenges to normal deontic logics

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- If the miners are in shaft A, we ought to block shaft A.
- If the miners are in shaft B, we ought to block shaft B.
- We ought to block neither shaft.

- $s_A \lor s_B$
- $O(s_A \rightarrow b_A)$
- $O(s_B \rightarrow b_B)$
- $O(\neg b_A \land \neg b_B)$

In any normal modal logic, these entail

- $O(\neg s_A \land \neg s_B \land \neg b_A \land \neg b_B)$