

Logic 2: Modal Logic

Lecture 13

Wolfgang Schwarz

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University of Edinburgh

Review

$M, w \models OA$ iff $M, v \models A$ for all v with wRv .

$M, w \models PA$ iff $M, v \models A$ for some v with wRv .

What is R ?

- wRv iff all norms are respected at v .
- wRv iff all norms of w are respected at v .

Assuming seriality, the resulting deontic logic is either KD45 or D.

Kripke semantics for deontic logic faces many problems.

- Response 1: replace Kripke semantics by a different semantics.
- Response 2: explain away the problems.
 - Appeal to pragmatics.
 - Redefine the accessibility relation.
 - Offer a new translation for deontic conditionals.

Pragmatics

(1) Amy is at the pub or in the library.

↗ (1a) Amy might be at the pub.

↗ (1b) Amy might be in the library.

This is not an entailment, but an **implicature**.

- It would (normally) be uncooperative to assert (1) unless (1a) and (1b) were true.
- So we take an assertion of (1) to indicate the truth of (1a) and (1b).

(2) All students got a First or a Second.

↪ (2a) Some students got a First.

↪ (2b) Some students got a Second.

Ross's Paradox

(3) You must either mail the letter or burn it.

↪ (3a) You are permitted to mail the letter.

↪ (3b) You are permitted to burn the letter.

The Paradox of Free Choice

(4) You may have beer or wine.

↪ (4a) You may have beer.

↪ (4b) You may have wine.

Signs that this is an implicature:

- ‘You may have beer or wine – I can’t tell you which.’
- ‘You may not have beer’ does not seem to imply ‘You may not have beer or wine’.

Redefining Accessibility

The Bank Robber Paradox

Mary robbed a bank.

1. Mary ought to go to jail. Oj
2. Mary ought to not have robbed the bank. $O \neg r$
3. But not: $O(j \wedge \neg r)$.

In an ideal world, Mary didn't rob the bank and doesn't go to jail.

Assuming wRv iff v is ideal (relative to w), then Oj is false.

Redefining Accessibility

Why is it true that Mary ought to go to jail?

Why is it better if Mary goes to jail than if she doesn't?

Intuitively: because $r \wedge j$ worlds are better than $r \wedge \neg j$ worlds.

If we hold fixed that Mary robbed the bank, then best worlds are worlds at which she goes to jail.

Equivalently:

Among worlds where Mary robbed the bank, the best worlds are worlds at which she goes to jail.

New definition of accessibility:

wRv iff v is one of the best worlds (by w 's norms) among those in which relevant circumstances from w obtain.

- O_j because Mary goes to jail in all the best worlds among those where she robbed the bank.

The new definition captures the fact that obligations depend not just on the norms, but also on the circumstances.

- Circumstance: Mary robbed a bank.
- Norm: Anyone who robs a bank must go to jail.
- So: Mary must go to jail.

Redefining Accessibility

The new definition captures the fact that obligations depend not just on the norms, but also on the circumstances.

- Circumstance: You walk past a drowning baby.
- Norm: Don't let babies drown.
- So: You must rescue the baby.

The Bank Robber Paradox

1. Mary ought to go to jail. Oj
2. Mary ought to not have robbed the bank. $O \neg r$
3. But not: $O(j \wedge \neg r)$.

Different circumstances are (pragmatically) held fixed in 1 and 2.

If we don't equivocate, either Oj or $O \neg r$ is false.

The Samaritan Paradox

1. Jones ought to help the injured Smith.
2. That Jones helps the injured Smith entails that Smith has been injured.
3. But not: Smith ought to have been injured.

Different circumstances are (pragmatically) held fixed in 1 and 3.

Professor Procrastinate

1. Professor Procrastinate ought not to accept the review.
2. Professor Procrastinate ought to accept and complete the review.

Different circumstances are held fixed in 1 and 2.

Deontic conditionals

(*) If John kills his mother, he should kill her gently.

Two obvious translations:

- $p \rightarrow Oq$ seems wrong: (*) and p do not entail Oq .
- $O(p \rightarrow q)$ seems wrong: $O\neg p$ entails $O(p \rightarrow r)$.

Deontic conditionals

(*) If John kills his mother, he should kill her gently.

Intuitively: (*) says that at the best worlds among those at which John kills his mother, he kills her gently.

This cannot be expressed in \mathcal{L}_M .

But we can add a **binary** obligation operator $O(\cdot/\cdot)$ to express it: $O(q/p)$.

$O(q)$ is true iff q is true at the best worlds at which relevant circumstances obtain.

$O(q/p)$ is true iff q is true at the best worlds at which relevant circumstances obtain and p is the case.