Review
Temporal operators:

- F: It will (at some point) be the case that
- G: It is always going to be the case that
- P: It was (at some point) the case that
- H: It has always been the case that
A **temporal model** consists of

- a non-empty set $T$ (of “times”),
- a binary relation $<$ on $T$ (the **precedence relation**),
- a function $V$ that assigns to each sentence letter of $\mathcal{L}_T$ and each member of $T$ a truth-value (1 or 0).
## Standard Temporal Semantics

| (g)  | $M, t \models FA$   | iff $M, s \models A$ for some $s$ such that $t < s$. |
| (h)  | $M, t \models GA$   | iff $M, s \models A$ for all $s$ such that $t < s$. |
| (i)  | $M, t \models PA$   | iff $M, s \models A$ for some $s$ such that $s < t$. |
| (j)  | $M, t \models HA$   | iff $M, s \models A$ for all $s$ such that $s < t$. |
Branching time
Is $F_s$ true at $t$? Yes.
Is $F\neg s$ true at $t$? Yes.

Intuition: ‘There will be a sea battle’ is not true at $t$. 
Branching time

Is $Fs$ true at $t$?

Intuition: 'There will be a sea battle' is not true at $t$. 
Is $F_s$ true at $t$? Yes.
Branching time

Is $F_s$ true at $t$? Yes.
Is $F \neg s$ true at $t$?
Is $Fs$ true at $t$? Yes.

Is $F \neg s$ true at $t$? Yes.
Is $F_{s}$ true at $t$? Yes.

Is $F_{\neg s}$ true at $t$? Yes.

Intuition: ‘There will be a sea battle’ is not true at $t$. 
Branching time

Standard semantics:

\[ M, t \models FA \text{ iff } M, r \models A \text{ for some } r \text{ such that } t < r \]

iff \( A \) is true at some future point on some history through \( t \).
Branching time

Standard semantics:
\[ M, t \models F A \iff M, r \models A \text{ for some } r \text{ such that } t < r \]
iff \( A \) is true at some future point on some history through \( t \).

“Peircean” semantics (CTL):
\[ M, t \models F A \iff A \text{ is true at some future point on every history through } t. \]
Branching time

Standard semantics:

\[ M, t \models F A \iff M, r \models A \text{ for some } r \text{ such that } t < r \]
iff \( A \) is true at some future point on some history through \( t \).

“Peircean” semantics (CTL):

\[ M, t \models F A \iff A \text{ is true at some future point on every history through } t. \]

This makes \( Fs \) and \( F \neg s \) both false at \( t \).
Branching time

Standard semantics:

\[ M, t \models FA \iff M, r \models A \text{ for some } r \text{ such that } t < r \]

iff \( A \) is true at some future point on some history through \( t \).
Branching time

Standard semantics:

\[ M, t \models F A \iff M, r \models A \text{ for some } r \text{ such that } t < r \]

iff \( A \) is true at some future point on some history through \( t \).

“Ockhamist” semantics (CTL*):

\[ M, h, t \models F A \iff A \text{ is true at some future point on history } h \text{ through } t. \]
Standard semantics:

\( M, t \models FA \) iff \( M, r \models A \) for some \( r \) such that \( t < r \)

iff \( A \) is true at some future point on some history through \( t \).

“Ockhamist” semantics (CTL*):

\( M, h, t \models FA \) iff \( A \) is true at some future point on history \( h \) through \( t \).

\( F s \) is true at \( t \) on the upper history, and false on the lower history.
Branching time

Standard semantics:

\[ M, t \models F A \iff A \text{ is true at some future point on some history through } t. \]

Peircean semantics:

\[ M, t \models F A \iff A \text{ is true at some future point on every history through } t. \]

Ockhamist semantics:

\[ M, h, t \models F A \iff A \text{ is true at some future point on history } h \text{ through } t. \]
Branching time

Standard semantics:

\[ M, t \models FA \iff A \text{ is true at some future point on some history through } t. \]

Peircean semantics:

\[ M, t \models FA \iff A \text{ is true at some future point on every history through } t. \]

Ockhamist semantics:

\[ M, h, t \models FA \iff A \text{ is true at some future point on history } h \text{ through } t. \]
\[ M, h, t \models \Diamond A \iff A \text{ is true at } t \text{ on some history through } t. \]
Branching time

Standard semantics:

\[ M, t \models FA \iff A \text{ is true at some future point on some history through } t. \]

Peircean semantics:

\[ M, t \models FA \iff A \text{ is true at some future point on every history through } t. \]

Ockhamist semantics:

\[ M, h, t \models FA \iff A \text{ is true at some future point on history } h \text{ through } t. \]
\[ M, h, t \models \Diamond A \iff A \text{ is true at } t \text{ on some history through } t. \]
\[ M, h, t \models \Diamond FA \iff A \text{ is true at some future point on some history through } t. \]
Branching time

Standard semantics:

\[ M, t \models F A \iff A \text{ is true at some future point on some history through } t. \]

Peircean semantics:

\[ M, t \models F A \iff A \text{ is true at some future point on every history through } t. \]

Ockhamist semantics:

\[ M, h, t \models F A \iff A \text{ is true at some future point on history } h \text{ through } t. \]
\[ M, h, t \models \Diamond A \iff A \text{ is true at } t \text{ on some history through } t. \]
\[ M, h, t \models \Diamond FA \iff A \text{ is true at some future point on some history through } t. \]
\[ M, h, t \models \Box FA \iff A \text{ is true at some future point on every history through } t. \]
• In Ockhamist semantics, truth is defined relative to three parameters: $M, h, t$. 

Supervaluationism: $M, h, t \models A$ if and only if $M, h, t \models A$ for all histories $h$ through $t$. 
• In Ockhamist semantics, truth is defined relative to three parameters: $M, h, t$.
• Only two of these represent a scenario and an interpretation: $M$ and $t$!
• In Ockhamist semantics, truth is defined relative to three parameters: $M, h, t$.
• Only two of these represent a scenario and an interpretation: $M$ and $t$!
• Ockhamism doesn’t tell us which sentences are true in a given scenario under a given interpretation.
• In Ockhamist semantics, truth is defined relative to three parameters: $M, h, t$. 
• Only two of these represent a scenario and an interpretation: $M$ and $t$! 
• Ockhamism doesn’t tell us which sentences are true in a given scenario under a given interpretation. 
• So it doesn’t tell us which sentences are true in all scenarios under all interpretations.
• In Ockhamist semantics, truth is defined relative to three parameters: $M, h, t$.
• Only two of these represent a scenario and an interpretation: $M$ and $t$.
• Ockhamism doesn’t tell us which sentences are true in a given scenario under a given interpretation.
• So it doesn’t tell us which sentences are true in all scenarios under all interpretations.
Branching time

• In Ockhamist semantics, truth is defined relative to three parameters: $M, h, t$.
• Only two of these represent a scenario and an interpretation: $M$ and $t$!
• Ockhamism doesn’t tell us which sentences are true in a given scenario under a given interpretation.
• So it doesn’t tell us which sentences are true in all scenarios under all interpretations.

Supervaluationism:

$M, t \models A$ iff $M, h, t \models A$ for all histories $h$ through $t$. 
Branching time

\[ M, t \models A \text{ iff } M, h, t \models A \text{ for all histories } h \text{ through } t. \]
\[ M, t \models A \text{ iff } M, h, t \models A \text{ for all histories } h \text{ through } t. \]

In supervaluationist Ockhamism,

- \( Fs \) and \( F \neg s \) are not true at \( t \).
Branching time

\[ M, t \models A \text{ iff } M, h, t \models A \text{ for all histories } h \text{ through } t. \]

In supervaluationist Ockhamism,

- \( Fs \) and \( F \neg s \) are not true at \( t \).
- \( \neg Fs \) and \( \neg F \neg s \) are not true at \( t \).
Branching time

$M, t \models A$ iff $M, h, t \models A$ for all histories $h$ through $t$.

In supervaluationist Ockhamism,

- $F_s$ and $F \neg s$ are not true at $t$.
- $\neg F_s$ and $\neg F \neg s$ are not true at $t$.
- $F_s$ and $F \neg s$ are neither true nor false at $t$. 
Branching time

\[ M, t \models A \text{ iff } M, h, t \models A \text{ for all histories } h \text{ through } t. \]

In supervaluationist Ockhamism,

- \( F_s \) and \( F \neg s \) are not true at \( t \).
- \( \neg F_s \) and \( \neg F \neg s \) are not true at \( t \).
- \( F_s \) and \( F \neg s \) are neither true nor false at \( t \).
- \( F_s \lor \neg F \neg s \) is true at \( t \).
If we look at possible scenarios, supervaluationist Ockhamism determines a three-valued logic: in any given scenario, a sentence can be

- true
- false
- neither
If we look at possible scenarios, supervaluationist Ockhamism determines a **three-valued logic**: in any given scenario, a sentence can be

- true
- false
- neither

The truth-value of a truth-functionally complex sentence at a scenario is not determined by the truth-value of the parts:
Branching time

If we look at possible scenarios, supervaluationist Ockhamism determines a **three-valued logic**: in any given scenario, a sentence can be

- true
- false
- neither

The truth-value of a truth-functionally complex sentence at a scenario is not determined by the truth-value of the parts:

- Fs and \( \neg F s \) are neither true nor false, \( Fs \lor \neg F s \) is true.
- Fs and \( F \neg \neg s \) are neither true nor false, \( Fs \lor F \neg \neg s \) is neither true nor false.
In other three-valued logics, the truth-value of truth-functionally complex sentences is determined by the truth-values of the parts:
In other three-valued logics, the truth-value of truth-functionally complex sentences is determined by the truth-values of the parts:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A ∨ B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>N</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Another application of supervaluationism: **vagueness**

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.
- We can assign a truth-value to $Fa$ relative to different sharpenings of $F$.
- But scenarios (and interpretations) don't include a sharpening.
- $M, w_j = Fa$ iff $M, w, s_j = Fa$ for all sharpenings $s$. 
Another application of supervaluationism: vagueness

• Suppose $F$ is a vague predicate, and $a$ is a borderline case.
Another application of supervaluationism: *vagueness*

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.
- We can assign a truth-value to $Fa$ relative to different *sharpenings* of $F$. 
Another application of supervaluationism: **vagueness**

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.
- We can assign a truth-value to $Fa$ relative to different **sharpenings** of $F$.
- $M, w, s \models Fa$
Another application of supervaluationism: **vagueness**

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.
- We can assign a truth-value to $Fa$ relative to different **sharpenings** of $F$.
- $M, w, s \models Fa$
- But scenarios (and interpretations) don’t include a sharpening.
Another application of supervaluationism: **vagueness**

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.
- We can assign a truth-value to $Fa$ relative to different **sharpenings** of $F$.
- $M, w, s \models Fa$
- But scenarios (and interpretations) don’t include a sharpening.
- $M, w \models Fa$ iff $M, w, s \models Fa$ for all sharpenings $s$. 
Another application of supervaluationism: 

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.

- $F_a$ is not true.
- $\neg F_a$ is not true.
- $F_a \lor \neg F_a$ is true.
Another application of supervaluationism: *vagueness*

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.
- $Fa$ is not true.
Another application of supervaluationism: vagueness

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.
- $Fa$ is not true.
- $\neg Fa$ is not true.
Another application of supervaluationism: **vagueness**

- Suppose $F$ is a vague predicate, and $a$ is a borderline case.
- $Fa$ is not true.
- $\neg Fa$ is not true.
- $Fa \lor \neg Fa$ is true.
Two-Dimensional Modal Logic
(*) Bob already knew yesterday that there would be a talk today.

Let $Y$ mean ‘yesterday’. 
(*) Bob already knew yesterday that there would be a talk today.

Let Y mean ‘yesterday’.

(*) cannot be translated as \( YK_b p \).
(⋆) Bob already knew yesterday that there would be a talk today.

Let Y mean ‘yesterday’.

(⋆) cannot be translated as Y K_b p.

- Y K_b p is true today iff K_b p is true yesterday.
Bob already knew yesterday that there would be a talk today.

Let Y mean ‘yesterday’.

(*) cannot be translated as $Y K_b p$.

- $Y K_b p$ is true today iff $K_b p$ is true yesterday.
- If $K_b p$ is true yesterday, then $p$ is true yesterday.
Bob already knew yesterday that there would be a talk today.

Let Y mean ‘yesterday’.

(*) cannot be translated as $Y K_b p$.

- $Y K_b p$ is true today iff $K_b p$ is true yesterday.
- If $K_b p$ is true yesterday, then $p$ is true yesterday.
- So $Y K_b p$ entails that $p$ is true yesterday.
(*) Bob already knew yesterday that there would be a talk today.

Let N mean ‘today’ (or ‘now’).
(*) Bob already knew yesterday that there would be a talk today.

Let N mean ‘today’ (or ‘now’).

Translation: $YK_b Np$. 
Two-Dimensional Modal Logic

What is the semantics of $N$?

This doesn't work:

$M, t_j = N$ iff $M, t_j = A$.

We want $Np$ to be true yesterday iff $p$ is true today.
What is the semantics of $N$?

This doesn’t work: $M, t \models N A$ iff $M, t \models A$.
What is the semantics of $N$?

This doesn’t work: $M, t \models NA$ iff $M, t \models A$.

We want $N p$ to be true yesterday iff $p$ is true today.
Sentences are true relative to two times. The first time parameter \((t_0)\) holds fixed the time of the scenario.

- \(YKbNp\) is true at \(t_1, t_2\) iff \(KbNp\) is true at \(t_1, t_2\).
- If \(KbNp\) is true at \(t_1, t_2\), then \(Np\) is true at \(t_1, t_2\).
- If \(Np\) is true at \(t_1, t_2\), then \(p\) is true at \(t_1, t_2\).
- So if \(YKbNp\) is true at \(t_1, t_2\) then \(p\) is true at \(t_1, t_2\).

Semantics of \(N\)

\[
M, t_0, t \models N\ A \iff M, t_0, t_0 \models A.
\]
Two-Dimensional Modal Logic

Semantics of $N$

$M, t_0, t \models N A \iff M, t_0, t_0 \models A.$

Sentences are true relative to two times.
Two-Dimensional Modal Logic

Semantics of $N$

| $M, t_0, t \models N A$ iff $M, t_0, t_0 \models A$. |

Sentences are true relative to two times.

The first time parameter ($t_0$) holds fixed the time of the scenario.
Two-Dimensional Modal Logic

<table>
<thead>
<tr>
<th>Semantics of $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M, t_0, t \models N A$ iff $M, t_0, t_0 \models A$.</td>
</tr>
</tbody>
</table>

Sentences are true relative to two times.

The first time parameter ($t_0$) holds fixed the time of the scenario.

- $YK_b N p$ is true at $t, t$ iff $K_b N p$ is true at $t, t^-$. 
Sentences are true relative to two times.

The first time parameter \(t_0\) holds fixed the time of the scenario.

- \(Y K_b N p\) is true at \(t, t\) iff \(K_b N p\) is true at \(t, t^-\).
- If \(K_b N p\) is true at \(t, t^-\), then \(Np\) is true at \(t, t^-\).
Two-Dimensional Modal Logic

<table>
<thead>
<tr>
<th>Semantics of N</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ M, t_0, t \models N A \text{ iff } M, t_0, t_0 \models A. ]</td>
</tr>
</tbody>
</table>

Sentences are true relative to two times.

The first time parameter \( t_0 \) holds fixed the time of the scenario.

- \( \text{Y} K_b N \ p \) is true at \( t, t \) iff \( K_b N \ p \) is true at \( t, t^- \).
- If \( K_b N \ p \) is true at \( t, t^- \), then \( N \ p \) is true at \( t, t^- \).
- If \( N \ p \) is true at \( t, t^- \), then \( p \) is true at \( t, t \).
Two-Dimensional Modal Logic

Semantics of $N$

\[ M, t_0, t \models N A \iff M, t_0, t_0 \models A. \]

Sentences are true relative to two times.

The first time parameter ($t_0$) holds fixed the time of the scenario.

- $Y K_b N p$ is true at $t, t$ iff $K_b N p$ is true at $t, t^-$.  
- If $K_b N p$ is true at $t, t^-$, then $N p$ is true at $t, t^-$.  
- If $N p$ is true at $t, t^-$, then $p$ is true at $t, t$.  
- So if $Y K_b N p$ is true at $t, t$ then $p$ is true at $t, t$.  

Two-Dimensional Modal Logic

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Expression</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$M, t_0, t \models \rho$</td>
<td>iff $V(\rho, t) = 1$.</td>
</tr>
<tr>
<td>(b)</td>
<td>$M, t_0, t \models \neg A$</td>
<td>iff $M, t_0, t \not\models A$.</td>
</tr>
<tr>
<td>(c)</td>
<td>$M, t_0, t \models A \land B$</td>
<td>iff $M, t_0, t \models A$ and $M, t_0, t \models B$.</td>
</tr>
<tr>
<td>(d)</td>
<td>$M, t_0, t \models A \lor B$</td>
<td>iff $M, t_0, t \models A$ or $M, t_0, t \models B$.</td>
</tr>
<tr>
<td>(g)</td>
<td>$M, t_0, t \models FA$</td>
<td>iff $M, s \models A$ for some $s \in T$ such that $t &lt; s$.</td>
</tr>
<tr>
<td>(h)</td>
<td>$M, t_0, t \models GA$</td>
<td>iff $M, s \models A$ for all $s \in T$ such that $t &lt; s$.</td>
</tr>
<tr>
<td>(k)</td>
<td>$M, t_0, t \models NA$</td>
<td>iff $M, t_0, t_0 \models A$.</td>
</tr>
</tbody>
</table>
### Two-Dimensional Temporal Semantics

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$M, t_0, t \models \rho$</td>
<td>iff $V(\rho, t) = 1$.</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>$M, t_0, t \models \neg A$</td>
<td>iff $M, t_0, t \not\models A$.</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>$M, t_0, t \models A \land B$</td>
<td>iff $M, t_0, t \models A$ and $M, t_0, t \models B$.</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>$M, t_0, t \models A \lor B$</td>
<td>iff $M, t_0, t \models A$ or $M, t_0, t \models B$.</td>
<td></td>
</tr>
<tr>
<td>(g)</td>
<td>$M, t_0, t \models FA$</td>
<td>iff $M, s \models A$ for some $s \in T$ such that $t &lt; s$.</td>
<td></td>
</tr>
<tr>
<td>(h)</td>
<td>$M, t_0, t \models GA$</td>
<td>iff $M, s \models A$ for all $s \in T$ such that $t &lt; s$.</td>
<td></td>
</tr>
<tr>
<td>(k)</td>
<td>$M, t_0, t \models NA$</td>
<td>iff $M, t_0, t_0 \models A$.</td>
<td></td>
</tr>
</tbody>
</table>

Scenarios don’t have two times. When is a sentence true at a time in a model?
Two-Dimensional Modal Logic

<table>
<thead>
<tr>
<th>Two-Dimensional Temporal Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $M, t_0, t \models \rho$ iff $V(\rho, t) = 1$.</td>
</tr>
<tr>
<td>(b) $M, t_0, t \models \neg A$ iff $M, t_0, t \not\models A$.</td>
</tr>
<tr>
<td>(c) $M, t_0, t \models A \land B$ iff $M, t_0, t \models A$ and $M, t_0, t \models B$.</td>
</tr>
<tr>
<td>(d) $M, t_0, t \models A \lor B$ iff $M, t_0, t \models A$ or $M, t_0, t \models B$.</td>
</tr>
<tr>
<td>(g) $M, t_0, t \models FA$ iff $M, s \models A$ for some $s \in T$ such that $t &lt; s$.</td>
</tr>
<tr>
<td>(h) $M, t_0, t \models GA$ iff $M, s \models A$ for all $s \in T$ such that $t &lt; s$.</td>
</tr>
<tr>
<td>(k) $M, t_0, t \models NA$ iff $M, t_0, t_0 \models A$.</td>
</tr>
</tbody>
</table>

Scenarios don’t have two times. When is a sentence true at a time in a model?

**Answer:** $M, t \models A$ iff $M, t, t \models A$. 

18
An interesting consequence:

\[ \neg j = 2p \]

\[ \neg \neg j = 2(\neg p) \]

So there are logical truths that will become false in the future!
An interesting consequence:

- $\mathcal{N} p \rightarrow p$
An interesting consequence:

- $\models Np \rightarrow p$
- $\not\models G(Np \rightarrow p)$

So there are logical truths that will become false in the future!
An interesting consequence:

- $\models Np \rightarrow p$
- $\not\models G(Np \rightarrow p)$
An interesting consequence:

- $\models Np \rightarrow p$
- $\not\models G(Np \rightarrow p)$

So there are logical truths that will become false in the future!