Logic 2: Modal Logic

Lecture 15

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Mention: multi-modal translations in test

Mention: points for blank answers

Conditionals in natural language

Conditionals in natural language

- If Labour wins the election, there will be a second Brexit referendum.
- If we don't reduce greenhouse gases, the climate might get out of control.
- If the patient has diabetes, the test will be positive.
- You will be faster if you take a taxi.
- If Jones hadn't untied the rope, Smith would not have fallen.
- Take another step and I'll knock you down.

Conditionals in natural language

Indicative:

- If Oswald did not kill Kennedy, someone else did.

Subjunctive/counterfactual:

- If Oswald had not killed Kennedy, someone else would have.

Material conditionals

Material conditionals

Intuitively,

- *If A then B* and *A* entail *B*.
- *If A then B* and not-*B* entail not-*A*.
- *If A then B* is not equivalent to *If B then A*.

A	B	if A then B
1	1	1
1	0	0
0	1	1
0	0	1

"Paradoxes of material implication":

1. $B \models A \rightarrow B$

   The lecture ends at 11am. Therefore: If the building collapses at 10.45 then the lecture ends at 11am.

2. $\neg(A \rightarrow B) \models A$

   It is not the case that if it will rain tomorrow then the Moon will fall onto the Earth. Therefore: It will rain tomorrow.

3. $A \rightarrow B \models \neg B \rightarrow \neg A$

   If our opponents are cheating, we will never find out. Therefore: If we will find out that our opponents are cheating, then they aren't cheating.

4. $A \rightarrow B \models (A \wedge C) \rightarrow \neg B$

   If you add sugar to your coffee, it will taste good. Therefore: If you add sugar and vinegar to your coffee, it will taste good.

Material conditionals

Edgington (2007):

1. If *A* entails *B*, then we should be at least as confident in *B* as in *A*.
2. I am confident that the lecture ends at 11am.
3. But I am not confident that if the building collapses at 10.45 then the lecture ends at 11am.

Material conditionals

So there are good reasons to think that English conditionals are not truth-functional.

This is obvious for subjunctive conditionals.

- If Oswald had not killed Kennedy, Kennedy would have been re-elected.
- If Oswald had not killed Kennedy, someone else would have.

Strict conditionals

Strict conditionals

Russell and Whitehead, *Principia Mathematica* (1913):

> *...if p and ¬p ∨ q are both true, then q is true. In this sense, the proposition ¬p ∨ q will be quoted as stating that p* implies *q. (p.7)*

So 'the building collapses at 10.45' implies 'the lecture ends at 11'.

C.I. Lewis (1918):

- ¬p ∨ q is not a good formalization of 'p implies q'.
- A better one is $\Box(p \to q)$.

Some have argued that $\Box(p \to q)$ is also a good formalization of 'if p then q'.

Define $A \dashrightarrow B$ as $\Box(A \rightarrow B)$.

Kripke semantics for $\dashrightarrow$

If $M = \langle W, R, V \rangle$ is a Kripke model, then

$M, w \models A \dashrightarrow B$ iff for all v such that wRv, either $M, v \not\models A$ or $M, v \models B$.

Strict conditionals

Good news:

- $B \not\models A \rightarrow_3 B$
- $\neg A \not\models A \rightarrow_3 B$
- $\neg(A \rightarrow_3 B) \not\models A$

Strict conditionals

Bad news:

- $A \dashv3 B \models \neg B \dashv3 \neg A$

  If our opponents are cheating, we will never find out. Therefore: If we will find out that our opponents are cheating, then they aren't cheating.

- $A \dashv3 B \models (A \land C) \dashv3 \neg B$

  If you add sugar to your coffee, it will taste good. Therefore: If you add sugar and vinegar to your coffee, it will taste good.

- $A \dashv3 B, B \dashv3 C \models A \dashv3 C$.

  If I quit my job, I won't be able to pay rent. If I win a million, I'll quit my job. Therefore: if I win a million, I won't be able to pay rent.

Strict conditionals

Possible response:

The accessibility relation depends on conversational context.

- 'If you add sugar to your coffee, it will taste good.'
  – Here worlds where you add sugar and vinegar to your coffee are ignored/inaccessible.
- 'If you add sugar and vinegar to your coffee, it will taste good.'
  – Now these worlds are no longer ignored/inaccessible.

Strict conditionals

Hypothesis: For indicative conditionals, *wRv* iff *v* is compatible with what is known at *w*.

- If Oswald did not kill Kennedy then someone else did.
- If you add sugar to your coffee, it will taste good.

Modus ponens and modus tollens are valid because knowledge is factive.

- Suppose $\Box(A \rightarrow B)$ and *A*.
- By factivity, $\Box(A \rightarrow B)$ entails $A \rightarrow B$.
- $A \rightarrow B$ and *A* entail *B*.
- So *B*.

Strict conditionals

Another problem: Why are we often unsure about indicative conditionals?

- I'm not sure that there will be another referendum if Labour wins the election.

This is not because I'm unsure about what I know.

Strict conditionals

What about subjunctive conditionals?

- If Oswald had not killed Kennedy, someone else would have.

Intuitively, to assess a subjunctive conditional, we

1. rewind the world to the time of the antecedent,
2. make minimal changes to render the antecedent true,
3. then let history run its course.

The conditional is true iff the consequent is true at all the resulting worlds.

We are often unsure about subjunctive conditionals because we lack information about the world.

Strict conditionals

Different antecedents call for different revisions to the actual world.

- If Oswald had not killed Kennedy ...
- If Marilyn Monroe had killed Kennedy ...
- If Kennedy had died as an infant ...

If A entails B, then $\Box(B \rightarrow C)$ entails $\Box(A \rightarrow C)$.

But

- If Oswald had not killed Kennedy then Kennedy would have been re-elected.

does not entail

- If Marilyn Monroe had killed Kennedy then Kennedy would have been re-elected.

Strict conditionals

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1.     □(A → B)      (w) (Ass.)
2.     □(B → C)      (w) (Ass.)
3.    ¬□(A → C)      (w) (Ass.)
4.       wRv            (3)
5.     ¬(A → C)      (v)   (3)
6.        A          (v)   (5)
6.       ¬C          (v)   (5)
7.      A → B        (v)  (1,4)
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8.   ¬A   (v)  (7) 9.      B        (v)   (7)
     x
                 10.     A → C      (v)  (2,4)
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        11.  ¬A  (v)  (10)     12.   C   (v)  (10)
             x                       x
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