Constant and variable domains

Constant and variable domains

There are different ways to interpret the language of modal predicate logic.

- In constant domain semantics, we assume that the domain of (individual) quantification is the same relative to all worlds.
- In variable domain semantics, different worlds have different individual domains.

Constant and variable domains

A **variable domain model** consists of

1. a set of worlds W,
2. an accessibility relation R on W,
3. for each world w, a set of individuals D_w,
4. an interpretation function V that assigns
    - to each name a member of D,
    - to each n-place predicate and world w a set of n-tuples from D_w.

Constant and variable domains

- Suppose we want to figure out if $\Box Fa$ is true at a world.
- We need to check if Fa is true at all accessible worlds.
- At some of these worlds, the individual denoted by a may fail to exist.
- We assume (with negative free logic) this makes Fa false at those worlds.

Constant and variable domains

In variable domain semantics, **BF**, **CBF**, and **NE** are invalid.

$$(\mathbf{BF})\quad \forall x\Box A \rightarrow \Box\forall x A$$

$$(\mathbf{CBF})\quad \Box\forall x A \rightarrow \forall x\Box A$$

$$(\mathbf{NE})\quad \forall x\Box\exists y(y = x)$$

- **BF** becomes valid iff we assume that everything that exists at a world v accessible from w also exists at w.
- **CBF** becomes valid iff we assume that everything that exists at a a world w also exists at any world accessible from w.

Constant and variable domains

In variable domain semantics, **BF**, **CBF**, and **NE** are invalid.

$$(\textbf{BF})\quad \forall x \Box A \to \Box \forall x A$$

$$(\textbf{CBF})\quad \Box \forall x A \to \forall x \Box A$$

$$(\textbf{NE})\quad \forall x \Box \exists y (y = x)$$

Their CK-proofs employ the rule of Universal Instantiation:

$\forall x A$
$\therefore A[c/x]$

This rule is invalid in free logic.

Individual Concept Semantics

Individual Concept Semantics

We have assumed that the only function of a name or variable is to pick out an individual.

This render's Leibniz' Law valid.

$$\frac{\begin{array}{l} A \\ b = c \end{array}}{A[c//b]}$$

Individual Concept Semantics

There seem to be clear counterexamples:

Hammurabi knows that Hesperus is visible in the evening sky. $\Box Vh$
Hesperus = Phosphorus. $h = p$

∴ Hammurabi knows that Phosphorus is visible in the evening sky. $\Box Vp$

Individual Concept Semantics

Russell's response:

"...proper names are usually really descriptions. That is to say, the thought in the mind of a person using a proper name correctly can only be expressed explicitly if we replace the proper name by a description."

$\Box\exists x(Hx \wedge \forall y(Hy \rightarrow x{=}y) \wedge Vx)$
$h = p.$
$\neg\Box\exists x(Px \wedge \forall y(Py \rightarrow x{=}y) \wedge Vx)$

The bullet-biting response:

Hammurabi really did know that Phosphorus is visible in the evening sky.

Follow-up problem:

- Hammurabi believed that Phosphorus is not visible in the evening sky.
- On the bullet-biting account, Hammurabi had inconsistent beliefs.
- We can’t use Kripke semantics to model inconsistent beliefs.

Individual Concept Semantics

Frege's response:

The function of a name is not just to pick out an individual.

An individual can play many roles.

Different names for the individual may be associated with different roles.

A name that is associated with a role picks out whoever plays that role at any given world.

- 'Hesperus' picks out the brightest body in the evening sky.
- 'Phosphorus' picks out the brightest body in the morning sky.
- At our world, the names pick out the same object.
- At other worlds, they pick out different objects.

Individual Concept Semantics

A **constant-domain individual concept model** consists of

1. a set of worlds W,
2. an accessibility relation R on W,
3. a set of individuals D,
4. an interpretation function V that assigns
   - to each name and world w a member of D,
   - to each n-place predicate and world w a set of n-tuples from D.

Individual Concept Semantics

Definition

(a) $M, w, g \models \phi t_1 \ldots t_n$ iff $\langle [t_1]^{M,w,g}, \ldots, [t_n]^{M,w,g} \rangle \in V(\phi)$.

(b) $M, w, g \models s = t$ iff $[s]^{M,w,g} = [t]^{M,w,g}$.

(c) $M, w, g \models \neg A$ iff $M, w, g \not\models A$.

$\vdots$

Leibniz' Law is no longer valid for modal sentences.

Individual Concept Semantics

Problems:

1. $\Box\exists xA \rightarrow \exists x\Box A$ becomes valid.
2. There are no complete proof procedures.
3. Might not help with some counterexamples.

"Mary Ann Evans is George Elliot, but Smith doesn't know that she is."

What is the individual concept expressed by 'Mary Ann Evans'? The role under which Evans is known to us? Or the role under which she is known to Smith? We may not even know that role.

Counterpart semantics

Counterpart semantics

An alternative to individual concept semantics is **counterpart semantics**.

Names pick out individuals.

$\Box Fa$ is true at w iff every counterpart of a at every accessible world is F.

A counterpart of an individual is an individual that closely resembles it in certain respects.

Different names can make different resemblance criteria salient.

Counterpart semantics

"Hammurabi believes that Hesperus is visible in the evening sky."

Let *w* be one of Hammurabi's belief worlds.

- At *w*, the brightest object in the evening sky is different from the brightest object in the morning sky.
- The brightest object in the evening sky at *w* resembles Venus/Hesperus/Phosphorus at our world in a certain respect.
- The brightest object in the morning sky at *w* resembles Venus/Hesperus/Phosphorus at our world in a different respect.

Counterpart semantics

Possible advantages of counterparts over individual concepts:

- Things can have more than one counterpart at a world.
- Counterparthood need not be transitive, symmetric, or reflexive.

In counterpart semantics, **T** does not correspond to reflexivity of the accessibility relation, **4** does not correspond to transitivity, etc.

David Lewis's original counterpart semantics determines a strange logic in which e.g. $\Box(A \wedge B) \to \Box A$ is invalid.