

# Mind, Matter, and Language

## Lecture 3: Frege's Referential Semantics

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# Review

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- We can use language to share information.
- This is possible because there is a conventional link between sentences and truth-conditions.

In itself, a string of noises or scribbles is neither true nor false.

The conventions of English link 'it is raining' with certain kinds of situations: those in which it is raining.

- People tend to utter the sounds only in that kind of situation.
- People tend to assume that such a situation obtains if someone utters the sounds.

These situations are the **conditions under which the sentence is true**, for the English-speaking community.

'It is raining' is true in English iff it is raining.

- A sentence's truth-conditions (in a language) are typically determined by its parts and syntactic composition.
- A compositional semantics specifies how the meaning of complex expressions is built from the meaning and arrangement of their parts.
- What is the meaning of 'Bob'?
- The referential approach: the meaning of 'Bob' is Bob.

The referential approach was first systematically developed by Gottlob Frege, who later went on to develop an influential alternative.

We'll look at Frege's referential account first.

## Frege's project

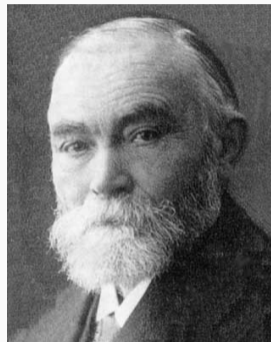
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## Frege's project

Gottlob Frege (1848-1925): German mathematician, logician, and philosopher.

Life project: to show that all truths of arithmetic and analysis are logically derivable from definitions.

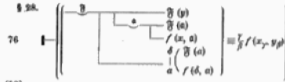
To give these derivations, Frege invented a formal language and proof system: the *Begriffsschrift*.



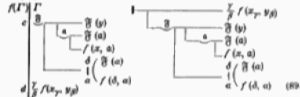


# Frege's project

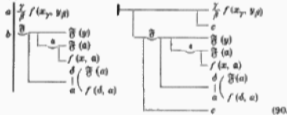
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(89) :



(90) :



69  
 $f / \exists$   
 $x y$   
 $\exists \Gamma$   $f(x, \Gamma)$   
 $d / \exists (a)$   
 $a \setminus f(d, a)$

(90) :



\*): In Bezug auf die Hühnung mit  $\exists$  siehe § 11.

*I first had to see how far one could get in arithmetic by means of inferences alone [...]. So that nothing intuitive could intrude here unnoticed, everything had to depend on the chain of inference free of gaps. In striving to fulfil this requirement in the strictest way, I found an obstacle in the inadequacy of language: however cumbersome the expressions that arose, the more complicated the relations became, the less the precision was attained that my purpose demanded. Out of this need came the idea of the present Begriffsschrift. It is thus intended to serve primarily to test in the most reliable way the validity of a chain of inference and reveal every presupposition that tends to slip in unnoticed, so that its origin can be investigated.*

Gottlob Frege, *Begriffsschrift* (1879)

## Use and mention

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Frege emphasized the distinction between a linguistic expression and its referent.

- The symbol '2' refers to the number 2.
- The word 'London' refers to the city London.
- 'London' has five letters. London has 8 million inhabitants.
- '2' is not a number. It is a name of a number.
- The Arabic numeral '2', the Roman numeral 'ii', the English word 'two', and the expression '1+1' all refer to the same number.
- What does "'2'" refer to?

### Philosophy jargon:

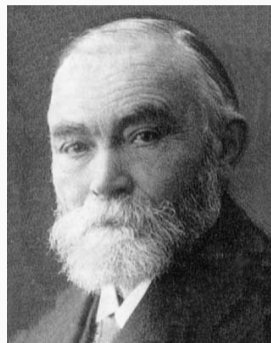
An expression outside quotation marks is **used**. An expression in quotation marks is **mentioned**.

- Science begins with curiosity.
- 'Science' begins with the nineteenth letter of the alphabet.

In the first sentence, the word 'Science' is used. In the second, it is mentioned.

*What is expressed in the equation [ $2 + 2 = 4$ ] is that the right-hand complex of signs has the same referent [Bedeutung] as the left-hand one. I must here combat the view that, e.g., [ $2 + 2$ ] and [ $4$ ] are equal but not the same. This view is grounded in the [...] confusion of form and content, sign and thing signified. It is as though one wanted to regard the sweet-smelling violet as differing from *viola odorata* because the names sound different.*

Frege, "Function and Concept" (1891)



# Functions

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Mathematicians often talk about functions.

- $2x^3 + x$
- $y = \sqrt{x} + 5$
- $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Frege: Functions are not sequences of symbols. They are what these sequences refer to.

' $2x^3 + x$ ' refers to a function that

- maps the number 1 to the number 3,
- maps the number 2 to the number 18,
- etc.



# Functions

To specify a function, you have to specify what it **returns** as **value** for any thing it can take as **argument**.

$2x^3 + x$  and  $\frac{d}{dx}8x^4 + \frac{x^2}{2} - 9$  are the same function.

The letter 'x' in ' $2x^3 + x$ ' does not refer to a particular number. It only indicates an argument place.

We could also write ' $2()^3 + ()$ '.

Frege calls function expressions **unsaturated**.

He also calls functions themselves unsaturated.

'the capital of Germany'

- 'the capital of Germany' refers to Berlin.
- 'the capital of Germany' is composed of 'the capital of' and 'Germany'.
- 'Germany' refers to Germany.
- What does 'the capital of' refer to?
- If we saturate 'the capital of' with a name for a country we get an expression that refers to a city.
- Frege: 'the capital of' is a function expression; it refers to a function.

'the capital of' refers to a function that maps

- Germany to Berlin,
- the UK to London,
- France to Paris,
- Madagascar to Antananarivo,
- ...

## Frege's referential semantics

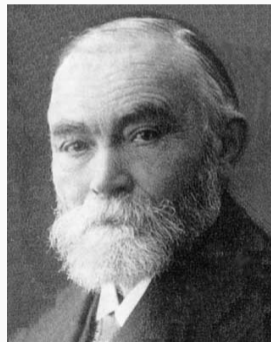
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## Frege's referential semantics

Guiding thought: to figure out what a word means, we need to look at how it contributes to the meaning of a sentence.

*Never ask for the meaning of a word in isolation,  
but only in the context of a sentence.*

Frege, *Foundations of Arithmetic* (1884)



'Bob smokes.'

- 'Bob' refers to Bob.
- What does 'smokes' refer to?
- Frege: 'smokes' is unsaturated; it refers to a function.

'smokes' refers to a function that maps people to **truth-values**:

- Bob to True,
- Alice to False,
- Carol to False,
- ...

Functions that return truth-values Frege calls **concepts**.

Fregean concepts are not mental entities; they are not ideas.



'Bob smokes.'

- 'Bob' refers to Bob.
- 'smokes' refers to a function that maps all and only the smokers to the truth-value True.
- A sentence composed of a functional part  $A$  and an argument part  $B$  is true iff the referent of  $A$  returns True for the referent of  $B$ .

'Somebody smokes.'

- What does 'somebody' refer to?
- 'smokes' refers to a function from people to truth-values. But 'somebody' doesn't refer to any particular person (who smokes and also doesn't smoke).
- Frege: 'somebody' is unsaturated; it refers to a function.

'somebody' refers to a **second-order function** that maps **first-order functions** to truth-values:

- $\llbracket \text{smokes} \rrbracket$  to True,
- $\llbracket \text{runs} \rrbracket$  to True,
- $\llbracket \text{owns all houses in Edinburgh} \rrbracket$  to False,
- $\llbracket \text{can jump over the moon} \rrbracket$  to False,
- ...

'Somebody smokes.'

- 'smokes' refers to a function that maps all and only the smokers to True.
- 'somebody' refers to a function that maps all and only the functions that map at least one person to True to True.
- A sentence composed of a functional part  $A$  and an argument part  $B$  is true iff the referent of  $A$  returns True for the referent of  $B$ .

Intuitively: 'somebody' is a predicate of predicates.

- 'Bob smokes' attributes to Bob the property of smoking.
- 'Somebody smokes' attributes to the property of smoking the property of having instances.

Similarly for 'nobody', 'everybody', 'something', 'nothing', 'everything', etc.

# The ontological argument

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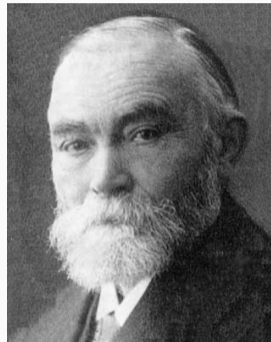
## The ontological argument for the existence of God:

- By definition, God is a being than which nothing greater can be conceived.
- All else equal, a being that has the property of existing is greater than one that doesn't.
- So any being than which nothing greater can be conceived exists.

## The ontological argument

*The ontological proof of God's existence suffers from the fallacy of treating existence as a first-level concept.*

Frege, "Function and Concept" (1891)





## The ontological argument

(1) Horses have four legs.

(2) Horses exist.

(3) Unicorns don't exist.

(1) attributes a property to horses.

(2) does not; it merely says that some things are horses.

(3) does not say that unicorns have an inferior property of not existing; it merely says that nothing is a unicorn.

### The ontological argument for the existence of God:

- By definition, God is a being than which nothing greater can be conceived.
- All else equal, a being that has the property of existing is greater than one that doesn't.
- So any being than which nothing greater can be conceived exists.

'God exists' does not attribute to God a superior property of existing.

'God doesn't exist' does not attribute to God an inferior property of not existing.